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USER'S MANUAL FOR LSTSQR-1

by

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Research Engineer

(The opinions, findings, and conclusions expressed in this report are those of the author and not necessarily those of the sponsoring agencies.)

Virginia Highway & Transportation Research Council
(A Cooperative Organization Sponsored Jointly by the Virginia Department of Highways & Transportation and the University of Virginia)

Charlottesville, Virginia

June 1976
VHTRC 76-R59

2822

ABSTRACT

This manual details the preparation of data for and the interpretation of output from the least squares computer program LSTSQR-1. The material presented here will be somewhat difficult for the non-computer oriented professional to interpret on the first reading. However, the professional researcher should find after an initial reading, some consultation with the data processing staff, and a little practice that he can easily design, prepare, and interpret his own data analyses.

LSTSQR-1 performs least squares curve fitting of data pairs under a wide variety of I/O and processing options. LSTSQR-1 accepts any number of consecutive data sets and any number of data points per data set. Data may come from any of six input files and may be read under any format specifying one data pair per record. The independent and dependent variables of the regression may be assigned and generated interchangeably and independently using a set of nine transformation functions each available in nineteen powers ranging from -9 to +9. Also the independent and dependent variable titles and the regression title may be supplied by the user. All regressions are polynomials in the independent variable of degree 1 through 9 with or without a constant term. Finally, the extent of output is under user control.

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LSTSQR-1

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INTRODUCTION

This report details the mechanics and philosophy of using LSTSQR-1, a computer program designed by W. A. Carpenter to perform a wide variety of regression analyses under a very flexible set of user controls. LSTSQR-1 and this manual provide the professional researcher with a valuable analytical tool.

The first section of this report details the command structure and input requirements of LSTSQR-1. The second section explains the formulation, meaning, and use of the regression statistics supplied by LSTSQR-1. Section three contains an example problem and program listing.

INPUT STRUCTURE

A LSTSQR-1 job for the CDC6400 computer (LSTSQR-1 is IBM compatible with appropriate JCL) consists of

1. Red Header Card,
2. REQUEST (TAPE7,*A*),
3. REQUEST (TAPE8,*A*),
4. ATTACH Cards for any files TAPE9 through TAPE13 which are to be used for auxillary data input,
5. FTN(L=0).,
6. LGO.,
7. Orange CR Data Card,
8. LSTSQR-1 Source Deck,

9. Orange CR Data Card,

10. LSTSQR-1 Data Deck,

11. Blue END Card

A LSTSQR-1 Data Deck consists of a sequence of data sets. Each such data set consists of command cards, specifying those options to be used, and the data (or the location of the data) to be analyzed.

Command Cards

Each data set consists of one or more command cards followed (optionally) by the data to be analyzed. The last command card in each data set must be either NEW or OLD. All other command cards are semi-optional. LSTSQR-1 is initialized (at compilation time) with a set of values for all command options. This set of options is then upgraded from data set to data set by means of command cards. Thus, for instance, if the LST command card is omitted from a data set, the list option remains unchanged from its previous value. (The user should note that omission of a command card does not generate a default to the initial value of that command but only to the previous value.) The valid commands and their parameters are given below.

Degree Command Card

The DEG card, format (A3,1X,I1), specifies the degree of the regression equation to be employed. The DEG card is structured as follows:

Columns 1 - 3, (A3): The alpha-string "DEG."

Column 5, (I1) : D, the degree to be used.
D must be $1 \le D \le 9$, otherwise the previous value of D will be used.

Independent Variable Command Card

The IND card, format (A3,2(1X,I1),1X,I2), specifies which variable in each input record (the first or second) is to be the independent variable, IV, how that variable is to be transformed, and what exponent is to be used in the transformation.

The transformation structure is

Transform Number, IN			Transformation
1	IV	=	VARIABLE(IK)**IP
2	IV	=	Log_e (VARIABLE(IK)**IP)
3	IV	=	Log_{10} (VARIABLE(IK)**IP)
4	IV	=	Exp (VARIABLE(IK)**IP)
5	IV	=	Exp(-(VARIABLE(IK)**IP)
6	IV	=	Sin (VARIABLE(IK)**IP)
7	IV	=	Cos (VARIABLE(IK)**IP)
8	IV	=	Sinh (VARIABLE(IK)**IP)
9	IV	=	Cosh (VARIABLE(IK)**IP)

The IND card is structured as follows:

Columns 1 - 3, (A3) :	The alpha-string "IND."
Column 5, (I1) :	IK, the variable number, IK must be $1 \le IK \le 2$, otherwise, the previous value of IK will be used.
Column 7, (I1) :	IN, the transformation number. IN must be $1 \le IN \le 9$, otherwise, the previous value of IN will be used.
Columns 9 - 10, (I2):	IP, the transformation power. IP must be $-9 \le IP \le 9$, otherwise, the previous value of IP will be used.

Dependent Variable Command Card

The DEP card, format (A3,2(1X,I1),1X,I2) specifies which variable in each input record (the first or second) is to be the dependent variable, DV, how that variable is to be transformed, and what exponent is to be used in the transformation. The transformation structure is

Transform Number, DN	Transformation
1	DV = VARIABLE(DK)**DP
2	DV = Log_e (VARIABLE(DK)**DP)
3	DV = Log_{10} (VARIABLE(DK)**DP)
4	DV = Exp (VARIABLE(DK)**DP)
5	DV = Exp (-(VARIABLE(DK)**DP))
6	DV = Sin (VARIABLE(DK)**DP)
7	DV = Cos (VARIABLE(DK)**DP)
8	DV = Sinh (VARIABLE(DK)**DP)
9	DV = Cosh (VARIABLE(DK)**DP)

The DEP card is structured as follows

Columns 1 - 3, (A3)	:	The alpha string "DEP."
Column 5, (I1)	:	DK, the variable number DK must be $1 \le DK \le 2$, otherwise the previous value of DK will be used.
Column 7, (I1)	:	DN, the transformation number. DN must be $1 \le DN \le 9$, otherwise the previous value of DN will be used.
Columns 9 - 10, (I2)	:	DP, the transformation power. DP must be $-9 \le DP \le 9$, otherwise the previous value of DP will be used.

Title Command Card

The TLE card, format (A3, T41,10A4), specifies the title (heading) to be placed on the output. The TLE card is structured as follows

Columns 1 - 3, (A3)	:	The alpha string "TLE."

Columns 41-80, (10A4) : A forty-character alpha string. Heading information should be centered in these forty columns.

Variable -1 Title Command Card

The ONE card, format (A3,T41,10A4), specifies the title (description) of the first variable on each data card. This information is output as part of LSTSQR-1 heading. The ONE card is structured as follows

Columns 1 - 3, (A3) : The alpha string "ONE."

Columns 41 - 80, (10A4) : A forty-character alpha string. Heading information should be centered in these forty columns.

Variable -2 Title Command Card

The TWO card, format (A3, T41,10A4), specifies the title (description) of the second variable on each data card. This information is output as part of LSTSQR-1 heading. The TWO card is structured as follows

Columns 1 - 3, (A3) : The alpha string "TWO."

Columns 41 - 80, (10A4) : A forty-character alpha string. Heading information should be centered in these forty columns.

Format Command Card

The FMT card, format (A3, T41,10A4), specifies the format under which the data (one data pair per record) are to be read. The FMT card is structured as follows

Columns 1 - 3, (A3) : The alpha string "FMT."

Columns 41 - 80, (10A4) : The data format. The format must be enclosed in parentheses and the word "format" must not appear. Only F, X, and T formats are allowed.

<u>Zero Command Card</u>

The ZRO card, format (A3,T12,L1), specifies whether or not the regression equation should be forced through the origin, i.e., through zero. The ZRO card is structured as follows

Columns 1 - 3, (A3)	:	The alpha string "ZRO."
Column 12, (L1)	:	"T" (force through zero) or "F" (do not force through zero).

<u>List Command Card</u>

The LST card, format (A3,T12,L1), specifies whether or not a table of actual vs. estimated dependent variables should be printed as part of the output. The LST card is structured as follows

Columns 1 - 3, (A3)	:	The alpha string "LST."
Column 12, (L1)	:	"T" (print the table) or "F" (do not print the table).

<u>New Data Command Card</u>

The NEW card, format (A3,1X,I1), specifies that data is to be read in for this analysis and also which file the data is on. The NEW card is structured as follows

Columns 1 - 3, (A3)	:	The alpha string "NEW."
Column 5, (I1)	:	F, the file on which the data resides. Note, there is a special default associated with F. If column 5 is left blank, F reverts to file 5, the card reader file. LSTSQR-1 has reserved files 9 through 13 as auxillary input files for the user. The user should consult a LSTSQR-1 source listing before using any input file other than the card file.

Old Data Command Card

The OLD card, format(A3), specifies that the data from the previous data set is to be used in this analysis. The OLD card is structured as follows

Columns 1 - 3, (A3) : The alpha string "OLD."

Initial Values of Command Card Parameters

The reader should recall that when defaults occur they reference the previous value of the defaulted variable, which is not necessarily the initial value. The initial parameter values are as follows.

DEG Card

D = 1

IND Card

IK = 1
IN = 1
IP = 1

DEP Card

DK = 2
DN = 1
DP = 1

TLE Card

TITLE = (ten blanks) "Title omitted by user" (nine blanks)

ONE Card

VARNAM(1) = (fifteen blanks) "Variable 1" (fifteen blanks)

TWO Card

VARNAM(2) = (fifteen blanks) "Variable 2" (fifteen blanks)

FMT Card

FORMAT ="(2E13.0)" (thirty-two blanks)

ZRO Card

ZERO = FALSE

LST Card

LIST = TRUE

Invalid Command Cards

If LSTSQR-1 finds a command card which does not have a valid three-character string in columns 1-3, it will output an error message and skip to the next accessible data set on file 5.

Data Cards

The data cards (or data records if data are on tape or disk) contain the data pairs to be analyzed as described by the command cards. If data are on cards, the data cards for the data set in question must immediately follow the NEW command card for the data set in question and must be terminated by an END card. (An END card is simply a card with the alpha string "END" in columns 1 - 3.) The command cards for the next data set will then immediately follow the END card. If the data for any data set is that from the previous data set, then an OLD command card must be used, and no data cards may appear in the data set. The command cards for the next data set will then immediately follow the OLD card. If the data for any data set is on a file other than the card file, file 5, then a NEW command card (with a file specified) must be used, and no data cards may appear in the data set. The command cards for the next data set will then immediately follow the NEW card.

LSTSQR-1 STATISTICAL OUTPUT

This section explains the formulation, meaning and usage of the statistics output by LSTSQR-1.

Definitions

Independent Variable, IV

As explained in the previous section, the independent variable is VARIABLE (IK) raised to the IP power and then transformed according to transform IN. LSTSQR-1 outputs (in the upper left-hand corner of page one of each data set) the description of VARIABLE (IK) and the equation relating IV and VARIABLE (IK).

Dependent Variable, DV

As explained in the previous section, the dependent variable is VARIABLE (DK) raised to the DP power and then transformed according to transform DN. LSTSQR-1 outputs (in the upper right-hand corner of page one of each data set) the description of VARIABLE (DK) and the equation relating DV and VARIABLE (DK).

Number of Data Points, N

LSTSQR-1 outputs (at the top center of page one of each data set) the number of data points used to perform the analysis.

Regression Equation and Degree, D & J

LSTSQR-1 lists (at the top center of page one of each data set) the polynomial regression equation used in the analysis. This equation is shown simply as

$$DV = \sum_{i=0}^{D} A_i \times (IV)^i,$$

where D is the degree of the regression equation. The regression coefficients A_0, $A_1 \ldots A_D$ are listed following an optional regression table. $J = D$ when $A_0 = 0$ (by using ZRO, TRUE command) and $J = D+1$ when A_0 is not forced to be zero.

Regression Estimate of Dependent Variable, DVE

The regression estimate of the dependent variable, DVE, corresponding to independent variable, IV, is found by substituting IV in the regression equation.

Regression Error, (DV-DVE)

The regression error (or estimation error) corresponding to the independent variable, IV, is found by evaluating (DV-DVE) corresponding to IV.

Average Squared Regression Error, AVG $[(\text{DV-DVE})^2]$

The average squared regression error is defined as

$$\text{AVG}\,[(\text{DV-DVE})^2] = \frac{1}{N} \sum_{i=1}^{N} (\text{DV-DVE})_i^2 .$$

Variance of Error of Estimate, VAR(DV-DVE)*

The variance of the error of estimate is defined as

$$\text{VAR(DV-DVE)} = \frac{1}{N-J} \sum_{i=1}^{N} (\text{DV-DVE})_i^2 .$$

Standard Error of Estimate, STD(DV-DVE)*

The standard error of estimate (the standard deviation of the regression error) is defined as

$$\text{STD (DV-DVE)} = \sqrt{\text{VAR (DV-DVE)}}$$

Variance of Independent Variable, VAR (IV)

The variance of the independent variable is defined as

$$\text{VAR(IV)} = \frac{N}{N-1} \left[\frac{1}{N} \sum_{i=1}^{N} (\text{IV})_i^2 - \left(\frac{1}{N} \sum_{i=1}^{N} (\text{IV})_i \right)^2 \right].$$

*Notice that these definitions are precipitated by the fact that AVG(DV-DVE) $\equiv$ 0 by virtue of the least squares regression process.

Standard Deviation of Independent Variable, STD(IV)

The standard deviation of the independent variable is defined as

$$STD(IV) = \sqrt{VAR(IV)}.$$

Variance of Dependent Variable VAR(DV)

The variance of the dependent variable is defined as

$$VAR(DV) = \frac{N}{N-1}\left[\frac{1}{N}\sum_{i=1}^{N}(DV)_i^2 - \left(\frac{1}{N}\sum_{i=1}^{N}(DV)_i\right)^2\right]$$

Standard Deviation of Dependent Variable, STD(DV)

The standard deviation of the dependent variable is defined as

$$STD(DV) = \sqrt{VAR(DV)}.$$

Index of Determination, R^2*

The index (coefficient) of determination is defined as

$$R^2 = 1 - \frac{VAR(DV-DVE) \times (N-J)}{VAR(DV) \times (N-1)}.$$

Index of Correlation, R*

The index (coefficient) of correlation is defined as $R = \sqrt{R^2}$.

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*R and R^2 are not calculated for the case in which the regression equation is forced through the origin.

Regression Covariance Matrix, M (i,j); i,j = 0,D

The regression covariance matrix is the inverse of the least-squares regression matrix. This matrix has the following properties.

VAR(DV-DVE) × M (i,i)	=	Variance of the i^{th} regression coefficient
STD (DV-DVE) × $\sqrt{M(i,i)}$	=	Standard deviation of the i^{th} regression coefficient
VAR(DV-DVE) × M(i,j)	-	Covariance between the i^{th} and j^{th} regression coefficients.

F1 and F2 Relationships

F (functional) relationships are characterized by having an underlying functional relationship between IV and DV. (eq. Hooke's Law, Velocity-Distance Relationships). When a functional relationship exists between IV and DV, R (and R^2) does not measure the correlation between IV and DV (implicitly IV and DV are completely correlated) but rather serves as a measure of how well the data group about the existing functional relation. Thus a small value of R does not imply that the functional relationship is incorrect, but rather that the data are excessively variable (poor technique or high experimental error).

S1 and S2 Relationships

S (statistical) relationships are characterized by a lack of an exact mathematical relationship between IV and DV (eq. Age vs. Income, Height vs. Weight). S1 relationships are those in which random samples are drawn from a population and two characteristics (X and Y or IV and DV) are measured. In S1 relationships either variable may be the dependent variable, and thus two regression equations are possible. For S1 relationships, R does indicate the extent of correlation between DV and f (IV) over the population where f(·) is the regression function. R is, of course, also a measure of how well the data group about f(·). S2 relationships differ from S1 relationships in that samples are not drawn randomly from a population but rather one of the variables is sampled only over a narrow range or at selected preassigned values such that it is not representative of the entire population. In S2 relationships the only regression equation which is valid is that in which IV is the restricted variable. R relative to S2 relationships measures the extent of correlation between DV and f(IV) over the restricted sample generated by IV (not over the population). Thus the statistician must use caution in analyzing the significance of R for S2 relationships. Also, as with all S and F relationships, R is a measure of how well the data group about f(·).

Statistical Tests

The 1-α confidence interval for the i^{th} regression coefficient, A_i, is given as

$$(A_i - t_{1-\alpha/2} \times STD(A_i), A_i + t_{1-\alpha/2} \times STD(A_i))$$

where $t_{1-\alpha/2}$ is found from a one-tailed t table with N-J degrees of freedom.

The 1-α confidence interval for DV corresponding to a fixed value, IV is given as

$$(DVE_{IV} - t_{1-\alpha/2} \times STD(DVE_{IV}), DVE_{IV} + t_{1-\alpha/2} \times STD\ (DVE_{IV}))$$

where $t_{1-\alpha/2}$is from a one-tailed t table with N-J degrees of freedome, DVE_{IV} = DVE corresponding to the chosen IV, and

$$STD(DVE_{IV}) = STD\ (DV-DVE) \times \sqrt{1 + L' \times M \times L}$$

where $L' = (1, IV^1, IV^2, \ldots IV^D)$ is the transpose of L, and M is the regression covariance matrix.

The 1-α confidence interval for $\tilde{DV}$ where $\tilde{DV}$ is the expected or average value of many experimental values of DV corresponding to a fixed value, IV, is given as

$$(DVE_{IV} - t_{1-\alpha/2} \times STD(\tilde{DVE}_{IV}), DVE_{IV} + t_{1-\alpha/2} \times STD(\tilde{DVE}_{IV}))$$

where all terms are as defined in the previous paragraph <u>except</u> that

$$STD(\tilde{DVE}_{IV}) = STD(DV-DVE) \times \sqrt{L' \times M \times L}.$$

(Note that the 1 under the radical is missing here since we have removed the intrinsic variability of DV by using $\tilde{DV}$.

The 1-α confidence band for the regression equation as a whole may be seen by plotting the two curves

$$f^{+}(X) = DVE_{X} + STD\,(DV\text{-}DVE) \times \sqrt{J \times F_{1-\alpha} \times \underline{X}' \times M \times \underline{X}}$$

$$f^{-}(X) = DVE_{X} - STD\,(DV\text{-}DVE) \times \sqrt{J \times F_{1-\alpha} \times \underline{X}' \times M \times \underline{X}}$$

where X varies over the range of interest in IV,

$\underline{X}'$ $= 1, X^{1}, X^{2} \ldots X^{D}$ is the transpose of $\underline{X}$, and

$F_{1-\alpha}$ is found from an F table with J and N-J degrees of freedom.

EXAMPLE PROBLEM AND LSTSQR-1 LISTING

Virginia interstate highway data are available on cards, Format (F2.0, 2X, F8.0, 2X, F5.0, 1X, F4.0, 2X, F4.0, 1X, F6.0), as Calendar Year, Million Vehicle Miles, Mean Speed, STD of Speed, Accidents, and Miles of Roadway. The analyses desired are Mean Speed vs. Calendar Year (linear); Accidents vs. Mean Speed (quadratic); and Accidents vs. STD of Speed (cubic). Figure 1 shows the inputs necessary to perform these analyses. Figure 2 shows the results of the analyses.

The remainder of this section contains a listing of LSTSQR-1 written in FORTRAN IV for application on a CDC 6400 machine.

R 324 REV.:
FEBRUARY 1975

VIRGINIA HIGHWAY & TRANSPORTATION RESEARCH COUNCIL

CARD CODE ENTRY FORM

PAGE 1 OF 1

NAME W.A. C. TEL. EXT. 313

PROJECT CHARGE NUMBER: 9903

SECTION NUMBER: 911

PROGRAM NAME, &/OR ANALYSIS, &/OR ACTION DESIRED: LSTSQR-1

DATE SUBMITTED: 5-4-76

NOTES	1–80
	TLE VIRGINIA INTERSTATE SYSTEM 65 THROUGH 73
	ONE CALENDAR YEAR
	TWO MEAN SPEED
	FMT (F2.0,2X,F5.0)
	NEW
Data Deck Here *	data cards
	END
	DEG 2
	ONE MEAN SPEED
	TWO ACCIDENTS
	FMT (14X,F5.0,7X,F4.0)
	OLD
	DEG 3
	ONE STD OF SPEED
	TWO ACCIDENTS
	FMT (20X,F4.0,2X,F4.0)
	OLD

FIGURE 1. LSTSQR-1 SAMPLE INPUT.

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LSTSQR -- VERSION1, JANUARY 1976          VIRGINIA INTERSTATE SYSTEM 65 THROUGH 73                    PAGE 1

THE INDEPENDENT VARIABLE, IV, IS                                                 THE DEPENDENT VARIABLE, DV, IS
          VAR1** 1                                                                          VAR2** 1
       WHERE VAR1 IS                                                                     WHERE VAR2 IS
        CALENDAR YEAR                                                                      MEAN SPEED

                              9 DATA POINTS WERE USED TO FIT THE EQUATION

                                   DV = SUM(A(I)*IV**I), I = 0, 1
```

IV INDEPENDENT VARIABLE	DV DEPENDENT VARIABLE	DVE REGRESSION ESTIMATE	DV-DVE REGRESSION ERROR
65.0000000000	59.4900000000	59.1253333333	.364666666678
66.0000000000	59.8500000000	60.0656666667	-.215666666658
67.0000000000	61.2000000000	61.0060000000	.194000000006
68.0000000000	62.0000000000	61.9463333333	.536666666703E-01
69.0000000000	62.6000000000	62.8866666667	-.286666666665
70.0000000000	63.8800000000	63.8270000000	.529999999985E-01
71.0000000000	63.7000000000	64.7673333333	-1.06733333334
72.0000000000	65.9900000000	65.7076666667	.282333333326
73.0000000000	67.2700000000	66.6480000000	.621999999991

Figure 2. LSTSQR-1 Sample output.

FIGURE 2 (CONT.)

LSTSQR -- VERSION1, JANUARY 1976 VIRGINIA INTERSTATE SYSTEM 65 THROUGH 73 PAGE 2

COEFFICIENT NAME	ESTIMATED VALUE	ESTIMATED VARIANCE	ESTIMATED STANDARD DEVIATION	DEGREES OF FREEDOM
A(0)	-1.99633333352	21.6905373386	4.65731009689	7
A(1)	.940333333336	.454950793651E-02	.674500403003E-01	7

*** REGRESSION STATISTICS ***
*** THE FOLLOWING STATISTICS ARE BASED ON DV AND IV, NOT VAR2 AND VAR1 ***

AVERAGE SQUARED REGRESSION ERROR	.212310370370
STANDARD (DEVIATION OF) ERROR OF ESTIMATE, STD(DV-DVE)	.522465765568
VARIANCE OF ERROR OF ESTIMATE, VAR(DV-DVE)	.272970476190
STANDARD DEVIATION OF DEPENDENT VARIABLE, STD(DV)	2.62117340138
VARIANCE OF DEPENDENT VARIABLE, VAR(DV)	6.87055000010
STANDARD DEVIATION OF INDEPENDENT VARIABLE, STD(IV)	2.73861278752
VARIANCE OF INDEPENDENT VARIABLE, VAR(IV)	7.49999999999
INDEX OF CORRELATION, R	.9825
INDEX OF DETERMINATION, R**2	.9652

REGRESSION COVARIANCE MATRIX, M(I,J), I,J = 0, 1

79.46111111	-1.150000000
-1.150000000	.1666666667E-01

THE ASSOCIATED F STATISTIC HAS 2 AND 7 DEGREES OF FREEDOM.

Figure 2 (cont.)

THE INDEPENDENT VARIABLE, IV, IS
VAR1** 1
WHERE VAR1 IS
MEAN SPEED

THE DEPENDENT VARIABLE, DV, IS
VAR2** 1
WHERE VAR2 IS
ACCIDENTS

9 DATA POINTS WERE USED TO FIT THE EQUATION

DV = SUM(A(I)*IV**I), I = 0, 2

IV INDEPENDENT VARIABLE	DV DEPENDENT VARIABLE	DVE REGRESSION ESTIMATE	DV-DVE REGRESSION ERROR
59.4900000000	3662.00000000	3481.20477366	180.795226344
59.8500000000	4111.00000000	3826.64505724	284.354942761
61.2000000000	4416.00000000	5060.18509431	-644.185094306
62.0000000000	5373.00000000	5745.08082979	-372.080829785
62.6000000000	6199.00000000	6236.24309613	-37.2430961318
63.8800000000	6729.00000000	7219.57147190	-490.571471896
63.7000000000	8133.00000000	7086.59673808	1046.40326192
65.9900000000	9005.00000000	8648.85010945	356.149890549
67.2700000000	9076.00000000	9399.62282949	-323.622829489

FIGURE 2 (CONT.)

LSTSQR -- VERSION1, JANUARY 1976 VIRGINIA INTERSTATE SYSTEM 65 THROUGH 73 PAGE 2

COEFFICIENT NAME	ESTIMATED VALUE	ESTIMATED VARIANCE	ESTIMATED STANDARD DEVIATION	DEGREES OF FREEDOM
A(0)	-149013.125093	18577159429.1	136298.053651	6
A(1)	4157.51817433	18596699.3792	4312.38905703	6
A(2)	-26.7970657872	1160.71330916	34.0692428615	6

*** REGRESSION STATISTICS ***
*** THE FOLLOWING STATISTICS ARE BASED ON DV AND IV, NOT VAR2 AND VAR1 ***

AVERAGE SQUARED REGRESSION ERROR	248393.879008
STANDARD (DEVIATION OF) ERROR OF ESTIMATE, STD(DV-DVE)	610.402177676
VARIANCE OF ERROR OF ESTIMATE, VAR(DV-DVE)	372590.818512
STANDARD DEVIATION OF DEPENDENT VARIABLE, STD(DV)	2083.77302693
VARIANCE OF DEPENDENT VARIABLE, VAR(DV)	4342110.02778
STANDARD DEVIATION OF INDEPENDENT VARIABLE, STD(IV)	2.62117340138
VARIANCE OF INDEPENDENT VARIABLE, VAR(IV)	6.87055000010
INDEX OF CORRELATION, R	.9673
INDEX OF DETERMINATION, R**2	.9356

REGRESSION COVARIANCE MATRIX, M(I,J), I,J = 0, 2

49859.41281	-1577.238450	12.45391557
-1577.238450	49.91185626	-.3942476388
12.45391557	-.3942476388	.3115249361E-02

THE ASSOCIATED F STATISTIC HAS 3 AND 6 DEGREES OF FREEDOM.

FIGURE 2 (CONT.)

LSTSQR -- VERSION1, JANUARY 1976

VIRGINIA INTERSTATE SYSTEM 65 THROUGH 73

THE INDEPENDENT VARIABLE, IV, IS
VAR1** 1
WHERE VAR1 IS
STD OF SPEED

THE DEPENDENT VARIABLE, DV, IS
VAR2** 1
WHERE VAR2 IS
ACCIDENTS

9 DATA POINTS WERE USED TO FIT THE EQUATION

DV = SUM(A(I)*IV**I), I = 0, 3

IV INDEPENDENT VARIABLE	DV DEPENDENT VARIABLE	DVE REGRESSION ESTIMATE	DV-DVE REGRESSION ERROR
5.89000000000	3662.00000000	5499.85857037	-1837.85857037
6.30000000000	4111.00000000	4050.78807199	60.2119280100
6.02000000000	4416.00000000	4328.33014876	87.6698512435
5.69000000000	5373.00000000	7435.72317204	-2062.72317204
5.24000000000	6199.00000000	5823.11860257	375.881397426
5.33000000000	6729.00000000	7328.43866593	-599.438665926
5.42000000000	8133.00000000	8116.59335759	16.4066424072
5.81000000000	9005.00000000	6312.75044030	2692.24955970
5.64000000000	9076.00000000	7808.39897028	1267.60102972

FIGURE 2 (CONT.)

COEFFICIENT NAME	ESTIMATED VALUE	ESTIMATED VARIANCE	ESTIMATED STANDARD DEVIATION	DEGREES OF FREEDOM
A(0)	-5670758.54470	.191613784004E+14	4377371.17462	5
A(1)	2934947.86423	.520722329615E+13	2281934.11302	5
A(2)	-504030.577156	156596905089.	395723.268319	5
A(3)	28753.0400104	521157877.798	22828.8825350	5

*** REGRESSION STATISTICS ***

*** THE FOLLOWING STATISTICS ARE BASED ON DV AND IV, NOT VAR2 AND VAR1 ***

AVERAGE SQUARED REGRESSION ERROR	1888862.80751
STANDARD (DEVIATION OF) ERROR OF ESTIMATE, STD(DV-DVE)	1843.89616126
VARIANCE OF ERROR OF ESTIMATE, VAR(DV-DVE)	3399953.05351
STANDARD DEVIATION OF DEPENDENT VARIABLE, STD(DV)	2083.77302693
VARIANCE OF DEPENDENT VARIABLE, VAR(DV)	4342110.02778
STANDARD DEVIATION OF INDEPENDENT VARIABLE, STD(IV)	.342969062422
VARIANCE OF INDEPENDENT VARIABLE, VAR(IV)	.117627777778
INDEX OF CORRELATION, R	.7146
INDEX OF DETERMINATION, R**2	.5106

REGRESSION COVARIANCE MATRIX, M(I,J), I,J = 0, 3

5635777.347	-2937649.160	509279.9970	-29364.90067
-2937649.159	1531557.411	-265569.1455	15315.71603
509279.9967	-265569.1454	46058.54923	-2656.798639
-29364.90065	15315.71602	-2656.798638	153.2838453

THE ASSOCIATED F STATISTIC HAS 4 AND 5 DEGREES OF FREEDOM.

THE LAST DATA SET HAS BEEN PROCESSED. END OF PROGRAM LSTSQR-1.

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```
      PROGRAM LSTSQR1(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE7,TAPE8, LSQ0000
     -TAPE9,TAPE10,TAPE11,TAPE12,TAPE13)                                 LSQ0010
C                                                                         C
C*************************************************************************C
C                                                                         C
C         LSTSQR -- VERSION 1 PERFORMS LEAST SQUARES CURVE FITTING OF     C
C         DATA PAIRS UNDER A VARIETY OF TRANSFORMATION CONFIGURATIONS.    C
C         THE LSTSQR-1 USER"S MANUAL SHOULD BE CONSULTED BEFORE ATTEMPTING C
C         TO USE THIS PROGRAM.  LSTSQR-1 WAS DEVELOPED FOR AND IS THE     C
C         PROPERTY OF THE VIRGINIA HIGHWAY AND TRANSPORTATION RESEARCH    C
C         COUNCIL BY WILLIAM A. CARPENTER, RESEARCH ENGINEER, OF THE DATA C
C         SYSTEMS AND ANALYSIS SECTION.  TEL:  804-977-0290               C
C                                                                         C
C*************************************************************************C
C                                                                         C
      INTEGER BLANK,DEG,DEGREE,DEP,DPOWER,DTRANS,DVAR,END,FILE,FMT,IND,  LSQ0020
     -IPOWER,ITRANS,IVAR,KEY,K1,K2,K3,LST,NEW,NPTS,OLD,ONE,PAGE,TLE,TWO, LSQ0030
     -ZRO                                                                LSQ0040
C                                                                         C
      REAL DV,DVE,ERR,IV,R,R2,SDDEZ,STDD,STDE,STDI,UNBIAS,VARD,VARDDE,   LSQ0050
     -VARI                                                               LSQ0060
C                                                                         C
      LOGICAL EVEN,K4,LIST,SINGULR,ZERO                                  LSQ0070
C                                                                         C
      INTEGER CARDIM(21),FORMAT(10),K5(10),MFMT(45),PIVOT(10),TITLE(10), LSQ0080
     -VARNAM(10,2)                                                       LSQ0090
C                                                                         C
      REAL A(10),DATA(2),MATRIX(10,21),SDV(11),SIV(19)                   LSQ0100
C                                                                         C
C                                                                         C
      DATA DEGREE/1/                                                     LSQ0110
C                                                                         C
      DATA IPOWER,DPOWER/1,1/                                            LSQ0120
C                                                                         C
      DATA ITRANS,DTRANS/1,1/                                            LSQ0130
C                                                                         C
      DATA IVAR,DVAR/1,2/                                                LSQ0140
C                                                                         C
      DATA ZERO,LIST/.FALSE.,.TRUE./                                     LSQ0150
C                                                                         C
      DATA DEG,DEP,END,FMT,IND,LST,NEW,OLD,ONE,TLE,TWO,ZRO/"DEG","DEP",  LSQ0160
     -"END","FMT","IND","LST","NEW","OLD","ONE","TLE","TWO","ZRO"/       LSQ0170
C                                                                         C
      DATA TITLE/2*"     ","  TI","TLE ","OMIT","TED ","BY U","SER ",2*" LSQ0180
     -    "/                                                             LSQ0190
C                                                                         C
      DATA VARNAM/3*"     ","   V","ARIA","BLE ","1   ",6*"     ","   V", LSQ0200
     -"ARIA","BLE ","2   ",3*"     "/                                    LSQ0210
C                                                                         C
      DATA BLANK,FORMAT/" ","(2F1","3.0)",8*"     "/                     LSQ0220
C                                                                         C
      DATA MFMT/                                                         LSQ0230
     -"(T51",",2(G","16.1","0,2X",")) ",                                 LSQ0240
     -"(T42",",3(G","16.1","0,2X",")) ",                                 LSQ0250
     -"(T33",",4(G","16.1","0,2X",")) ",                                 LSQ0260
     -"(T24",",5(G","16.1","0,2X",")) ",                                 LSQ0270
     -"(T15",",6(G","16.1","0,2X",")) ",                                 LSQ0280
```

```
     -"(I6,","/(G1","6.10",",2X)",")     ",
     -"(8(2","X,G1","4.8)",")     ","     ",
     -"(T5,","9(G1","2.6,","2X))","     ",
     -"(10(","1X,G","12.6",")) ","     "/
C
C
C
C        FILE 5 IS THE CARD READER.
C
 5000 FORMAT(A3,1X,I1,1X,I1,1X,I2,1X,L1,T41,10A4)
 5001 FORMAT(A3,19A4,A1)
C
C
C        FILE 6 IS THE LINE PRINTER.
C
 6000 FORMAT(1H1,"THE INVALID KEY WORD ** ",A3," ** WAS FOUND. PROGRAM W
     -ILL RESUME PROCESSING WITH THE DATA SET FOLLOWING THE NEXT *END* O
     -N FILE 5."/" CAUTION.......THIS ACTION MAY CAUSE ERRORS IF THIS JO
     -B CONTAINS DATA ON FILES OTHER THAN FIVE (5).")
 6001 FORMAT(1H1,T6,"LSTSQR -- VERSION1, JANUARY 1976",T47,10A4,T109,"PA
     -GE 1"//T6,"THE INDEPENDENT VARIABLE, IV, IS",T97,"THE DEPENDENT VA
     -RIABLE, DV, IS")
 6002 FORMAT(T18,"VAR",I1,"**",I2)
 6003 FORMAT(T15,"LOGE(VAR",I1,"**",I2,")")
 6004 FORMAT(T15,"LOG10(VAR",I1,"**",I2,")")
 6005 FORMAT(T16,"EXP(VAR",I1,"**",I2,")")
 6006 FORMAT(T14,"EXP(-(VAR",I1,"**",I2,"))")
 6007 FORMAT(1H+,T108,"VAR",I1,"**",I2)
 6008 FORMAT(1H+,T105,"LOGE(VAR",I1,"**",I2,")")
 6009 FORMAT(1H+,T105,"LOG10(VAR",I1,"**",I2,")")
 6010 FORMAT(1H+,T106,"EXP(VAR",I1,"**",I2,")")
 6011 FORMAT(1H+,T104,"EXP(-(VAR",I1,"**",I2,"))")
 6012 FORMAT(T15,"WHERE VAR",I1," IS",T105,"WHERE VAR",I1," IS"/T2,10A4,
     -T92,10A4//T41,I6," DATA POINTS WERE USED TO FIT THE EQUATION"//T52
     -,"DV = SUM(A(I)*IV**I), I = 0, ",I1)
 6013 FORMAT(T60,"WITH A(0) = 0")
 6014 FORMAT(T10,"THE LEAST SQUARES MATRIX FOR THIS PROBLEM WAS FOUND TO
     - BE SINGULAR, OR NEARLY SO."/T10,"PROGRAM WILL CONTINUE PROCESSING
     - WITH NEXT DATA SET.")
 6015 FORMAT(/T36,"IV",T56,"DV",T76,"DVE",T94,"DV-DVE"/T32, "INDEPENDENT
     -",T53,"DEPENDENT",2(10X,"REGRESSION")/T33,2("VARIABLE",12X),"ESTIM
     -ATE",T94,"ERROR"/)
 6016 FORMAT(T30,4(G18.12,2X))
 6017 FORMAT(1H1,T6,"LSTSQR -- VERSION1, JANUARY 1976",T47,10A4,T109,"PA
     -GE",1X,I2)
 6018 FORMAT(//T22,"COEFFICIENT",T37,3("ESTIMATED",11X),T97,"DEGREES  OF
     -"/T25,"NAME",T39,"VALUE",T58,"VARIANCE",T73,"STANDARD DEVIATION",
     -T99,"FREEDOM"/)
 6019 FORMAT(T25,"A(",I1,")",T34,3(G18.12,2X),T97,I6)
 6020 FORMAT(//T52,"*** REGRESSION STATISTICS ***")
 6021 FORMAT(T29,"*** THE FOLLOWING STATISTICS ARE BASED ON DV AND IV, N
     -OT VAR",I1," AND VAR",I1," ***"///T29,"AVERAGE SQUARED REGRESSION
     - ERROR",T89,G18.12//T29,"STANDARD (DEVIATION OF) ERROR OF ESTIMATE
     -, STD(DV-DVE)",T89,G18.12//T29,"VARIANCE OF ERROR OF ESTIMATE, VAR
     -(DV-DVE)",T89,G18.12//T29,"STANDARD DEVIATION OF DEPENDENT VARIABL
     -E, STD(DV)",T89,G18.12//T29,"VARIANCE OF DEPENDENT VARIABLE, VAR(D
     -V)",T89,G18.12//T29,"STANDARD DEVIATION OF INDEPENDENT VARIABLE, S
```

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     -TD(IV)",T89,G18.12//T29,"VARIANCE OF INDEPENDENT VARIABLE, VAR(IV) LSQ0770
     -",T89,G18.12//A1,T29,"INDEX OF CORRELATION, R",T89,F6.4//T29,"INDE LSQ0780
     -X OF DETERMINATION, R**2",T89,F6.4)                                  LSQ0790
 6022 FORMAT(/T43,"REGRESSION COVARIANCE MATRIX, M(I,J), I,J = 0, ",I1/) LSQ0800
 6023 FORMAT(/T38,"THE ASSOCIATED F STATISTIC HAS ",I1," AND",I3," DEGRE LSQ0810
     -ES OF FREEDOM.")                                                     LSQ0820
 6024 FORMAT(T34,"A MINIMUM OF ",I1," DATA POINTS ARE REQUIRED TO PERFOR LSQ0830
     -M THIS REGRESSION."/T24,"ONLY "I1," DATA POINTS WERE APPLIED.  PRO LSQ0840
     -GRAM WILL CONTINUE PROCESSING WITH THE NEXT DATA SET.")              LSQ0850
 6025 FORMAT(1X)                                                           LSQ0860
 6026 FORMAT(1H1,"THE LAST DATA SET HAS BEEN PROCESSED. END OF PROGRAM L LSQ0870
     -STSQR-1.")                                                           LSQ0880
 6027 FORMAT(T16,"SIN(VAR",I1,"**",I2,")")                                 LSQ0890
 6028 FORMAT(T16,"COS(VAR",I1,"**",I2,")")                                 LSQ0900
 6029 FORMAT(T15,"SINH(VAR",I1,"**",I2,")")                                LSQ0910
 6030 FORMAT(T15,"COSH(VAR",I1,"**",I2,")")                                LSQ0920
 6031 FORMAT(1H+,T106,"SIN(VAR",I1,"**",I2,")")                            LSQ0930
 6032 FORMAT(1H+,T106,"COS(VAR",I1,"**",I2,")")                            LSQ0940
 6033 FORMAT(1H+,T105,"SINH(VAR",I1,"**",I2,")")                           LSQ0950
 6034 FORMAT(1H+,T105,"COSH(VAR",I1,"**",I2,")")                           LSQ0960
C                                                                                C
C                                                                                C
C        FILE 7 IS A SCRATCH DISK CONTAINING CARD IMAGE DATA.                    C
C                                                                                C
C                                                                                C
C        FILE 8 IS A SCRATCH DISK CONTAINING TRANSFORMED DATA.                   C
C                                                                                C
 8000 FORMAT(2E21.14)                                                      LSQ0970
C                                                                                C
C                                                                                C
C                                                                                C
C                                                                                C
C                                                                                C
C        ** START OF LSTSQR-1 CODE **                                            C
C                                                                                C
C                                                                                C
C                                                                                C
C                                                                                C
 1    READ (5,5000) KEY,K1,K2,K3,K4,K5                                     LSQ0980
      IF(EOF(5))  999,2                                                    LSQ0990
C                                                                                C
C        TRANSFER ON VALUE OF KEY.                                               C
C                                                                                C
 2    IF(KEY .EQ. NEW) GO TO 10                                            LSQ1000
      IF(KEY .EQ. OLD) GO TO 20                                            LSQ1010
      IF(KEY .EQ. DEG) GO TO 30                                            LSQ1020
      IF(KEY .EQ. IND) GO TO 40                                            LSQ1030
      IF(KEY .EQ. DEP) GO TO 50                                            LSQ1040
      IF(KEY .EQ. TLE) GO TO 60                                            LSQ1050
      IF(KEY .EQ. ONE) GO TO 70                                            LSQ1060
      IF(KEY .EQ. TWO) GO TO 80                                            LSQ1070
      IF(KEY .EQ. FMT) GO TO 90                                            LSQ1080
      IF(KEY .EQ. ZRO) GO TO 100                                           LSQ1090
      IF(KEY .EQ. LST) GO TO 110                                           LSQ1100
C                                                                                C
C                                                                                C
C                                                                                C
```

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```
C                                                                           C
C        TO GET HERE THE KEY WAS INVALID. SKIP TO THE NEXT DATA SET.        C
C                                                                           C
      WRITE (6,6000) KEY                                              LSQ1110
C                                                                           C
 3    READ (5,5001) KEY                                               LSQ1120
      IF(EOF(5)) 999,4                                                LSQ1130
 4    IF(KEY-END) 3,1,3                                               LSQ1140
C                                                                           C
C                                                                           C
C                                                                           C
C     ** KEY .EQ. NEW **                                                    C
C        THIS IMPLIES THAT THERE ARE NO MORE KEYS FOR THIS DATA SET AND     C
C        THAT THE DATA FOLLOWS ON FILE K1.                                  C
C                                                                           C
 10   FILE=K1                                                         LSQ1150
      IF(FILE .EQ. 0) FILE=5                                          LSQ1160
      GO TO 200                                                       LSQ1170
C                                                                           C
C                                                                           C
C                                                                           C
C     ** KEY .EQ. OLD **                                                    C
C        THIS IMPLIES THAT THERE ARE NO MORE KEYS FOR THIS DATA SET AND     C
C        THAT THE DATA FROM THE PREVIOUS SET IS TO BE USED.                 C
C                                                                           C
 20   GO TO 300                                                       LSQ1180
C                                                                           C
C                                                                           C
C                                                                           C
C     ** KEY .EQ. DEG **                                                    C
C        IF K1 IS VALID, USE IT.  OTHERWISE USE THE LAST GIVEN VALUE.       C
C                                                                           C
 30   IF(K1 .GE. 1 .AND. K1 .LE. 9) DEGREE = K1                       LSQ1190
      GO TO 1                                                         LSQ1200
C                                                                           C
C                                                                           C
C                                                                           C
C     ** KEY .EQ. IND **                                                    C
C        IF K1, K2, AND 53 ARE VALID, USE THEM.  OTHERWISE USE THE          C
C        LAST GIVEN VALUES.                                                 C
C                                                                           C
 40   IF(K1 .EQ. 1 .OR. K1 .EQ. 2) IVAR = K1                          LSQ1210
      IF(K2 .GE.1 .AND. K2 .LE. 9) ITRANS = K2                        LSQ1220
      IF(K3 .GE. -9 .AND. K3 .LE. 9) IPOWER = K3                      LSQ1230
      GO TO 1                                                         LSQ1240
C                                                                           C
C                                                                           C
C                                                                           C
C                                                                           C
C     ** KEY .EQ. DEP **                                                    C
C        IF K1, K2, AND K3 ARE VALID, USE THEM.  OTHERWISE USE THE          C
C        LAST GIVEN VALUES.                                                 C
C                                                                           C
 50   IF(K1 .EQ. 1 .OR. K1 .EQ. 2) DVAR = K1                          LSQ1250
      IF(K2 .GE.1 .AND. K2 .LE. 9) DTRANS =K2                         LSQ1260
      IF( K3 .GE. -9 .AND. K3 .LE. 9) DPOWER = K3                     LSQ1270
      GO TO 1                                                         LSQ1280
```

```
C
C     ** KEY .EQ. TLE **
C        STORE THE TITLE.
C
 60   DO 61  I=1,10
      TITLE(I) = K5(I)
 61   CONTINUE
      GO TO 1
C
C
C
C     ** KEY .EQ. ONE **
C        STORE VARIABLE 1 NAME.
C
 70   DO 71  I=1,10
      VARNAM(I,1)=K5(I)
 71   CONTINUE
      GO TO 1
C
C
C
C     ** KEY .EQ. TWO **
C        STORE VARIABLE 2 NAME.
C
 80   DO 81 I=1,10
      VARNAM(I,2)=K5(I)
 81   CONTINUE
      GO TO 1
C
C
C
C     ** KEY .EQ. FMT **
C        STORE DATA INPUT FORMAT.
C
 90   DO 91   I=1,10
      FORMAT(I)=K5(I)
 91   CONTINUE
      GO TO 1
C
C
C
C     ** KEY .EQ. ZRO **
C
 100  ZERO=K4
      GO TO 1
C
C
C
C
C     ** KEY .EQ. LST **
C
 110  LIST = K4
      GO TO 1
C
C
C
C
```

```
C
C         READ IN NEW DATA.
C
 200  REWIND 7
      NPTS=0
 201  READ (FILE,5001) CARDIM
      IF(EOF(FILE))210,202
 202  IF(CARDIM(1).EQ.END) GO TO 210
      WRITE(7,5001) CARDIM
      NPTS= NPTS+1
      GO TO 201
 210  END FILE 7
 300  REWIND 7
      REWIND 8
      WRITE (6,6001) TITLE
C
C         WRITE THE EQUATION FORMS OF THE VARIABLES.
C
      GO TO (301,302,303,304,305,306,307,308,309),ITRANS
C
 301  WRITE (6,6002) IVAR,IPOWER
      GO TO 310
 302  WRITE (6,6003) IVAR,IPOWER
      GO TO 310
 303  WRITE (6,6004) IVAR,IPOWER
      GO TO 310
 304  WRITE (6,6005) IVAR,IPOWER
      GO TO 310
 305  WRITE (6,6006) IVAR,IPOWER
      GO TO 310
 306  WRITE (6,6027) IVAR,IPOWER
      GO TO 310
 307  WRITE (6,6028) IVAR,IPOWER
      GO TO 310
 308  WRITE (6,6029) IVAR,IPOWER
      GO TO 310
 309  WRITE (6,6030) IVAR,IPOWER
C
 310  GO TO (311,312,313,314,315,316,317,318,319),DTRANS
C
 311  WRITE (6,6007) DVAR,DPOWER
      GO TO 320
 312  WRITE (6,6008) DVAR,DPOWER
      GO TO 320
 313  WRITE (6,6009) DVAR,DPOWER
      GO TO 320
 314  WRITE (6,6010) DVAR,DPOWER
      GO TO 320
 315  WRITE (6,6011) DVAR,DPOWER
      GO TO 320
 316  WRITE (6,6031) DVAR,DPOWER
      GO TO 320
 317  WRITE (6,6032) DVAR,DPOWER
      GO TO 320
 318  WRITE (6,6033) DVAR,DPOWER
      GO TO 320
 319  WRITE (6,6034) DVAR,DPOWER
```

```
 320  WRITE (6,6012) IVAR,DVAR,(VARNAM(I,IVAR),I=1,10),(VARNAM(I,DVAR),I LSQ1970
     -=1,10),NPTS,DEGREE                                                  LSQ1980
C                                                                               C
C                                                                               C
      J=DEGREE+1                                                          LSQ1990
      J1=J+1                                                              LSQ2000
      J2=2*DEGREE+1                                                       LSQ2010
C                                                                               C
      IF(ZERO) WRITE (6,6013)                                             LSQ2020
      IF(.NOT.ZERO) WRITE (6,6025)                                        LSQ2030
C                                                                               C
      DO 321 I = 1,J2                                                     LSQ2040
      SIV(I)=0.0                                                          LSQ2050
 321  CONTINUE                                                            LSQ2060
C                                                                               C
      DO 322 I=1,J                                                        LSQ2070
      SDV(I)=0.0                                                          LSQ2080
 322  CONTINUE                                                            LSQ2090
      SDV(11)=0.0                                                         LSQ2100
C                                                                               C
C                                                                               C
C        READ THE DATA FROM SCRATCH DISK 7, TRANSFORM IT, AND GENERATE          C
C        SIV(I) AND SDV(I).                                                     C
C        SIV(I) CONTAINS THE SUM(IV**I-1), I=1,2*DEGREE+1                       C
C        SDV(I) CONTAINS THE SUM(DV*IV**I-1), I=1,DEGREE+1                      C
C        SDV(11) CONTAINS THE SUM(DV**2)                                        C
C                                                                               C
 330  READ (7,FORMAT) DATA                                                LSQ2110
      IF(EOF(7))370,340                                                   LSQ2120
C                                                                               C
C        GENERATE IV BY TRANSFORMING DATA(IVAR).                                C
C                                                                               C
 340  GO TO (341,342,343,344,345,346,347,348,349),ITRANS                  LSQ2130
C                                                                               C
 341  IV=DATA(IVAR)**IPOWER                                               LSQ2140
      GO TO 350                                                           LSQ2150
 342  IV=IPOWER*ALOG(DATA(IVAR))                                          LSQ2160
      GO TO 350                                                           LSQ2170
 343  IV=IPOWER*ALOG10(DATA(IVAR))                                        LSQ2180
      GO TO 350                                                           LSQ2190
 344  IV=EXP(DATA(IVAR)**IPOWER)                                          LSQ2200
      GO TO 350                                                           LSQ2210
 345  IV=EXP(-(DATA(IVAR)**IPOWER))                                       LSQ2220
      GO TO 350                                                           LSQ2230
 346  IV=SIN(DATA(IVAR)**IPOWER)                                          LSQ2240
      GO TO 350                                                           LSQ2250
 347  IV=COS(DATA(IVAR)**IPOWER)                                          LSQ2260
      GO TO 350                                                           LSQ2270
 348  IV=SINH(DATA(IVAR)**IPOWER)                                         LSQ2280
      GO TO 350                                                           LSQ2290
 349  IV=COSH(DATA(IVAR)**IPOWER)                                         LSQ2300
C                                                                               C
C        GENERATE DV BY TRANSFORMING DATA(DVAR).                                C
C                                                                               C
 350  GO TO (351,352,353,354,355,356,357,358,359),DTRANS                  LSQ2310
C                                                                               C
 351  DV=DATA(DVAR)**DPOWER                                               LSQ2320
```


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```
      GO TO 360                                                         LSQ02330
  352 DV=DPOWER*ALOG(DATA(DVAR))                                        LSQ02340
      GO TO 360                                                         LSQ02350
  353 DV=DPOWER*ALOG10(DATA(DVAR))                                      LSQ02360
      GO TO 360                                                         LSQ02370
  354 DV=EXP(DATA(DVAR)**DPOWER)                                        LSQ02380
      GO TO 360                                                         LSQ02390
  355 DV=EXP(-(DATA(DVAR)**DPOWER))                                     LSQ02400
      GO TO 360                                                         LSQ02410
  356 DV=SIN(DATA(DVAR)**DPOWER)                                        LSQ02420
      GO TO 360                                                         LSQ02430
  357 DV=COS(DATA(DVAR)**DPOWER)                                        LSQ02440
      GO TO 360                                                         LSQ02450
  358 DV=SINH(DATA(DVAR)**DPOWER)                                       LSQ02460
      GO TO 360                                                         LSQ02470
  359 DV=COSH(DATA(DVAR)**DPOWER)                                       LSQ02480
C                                                                              C
  360 WRITE (8,8000) IV,DV                                              LSQ02490
      DUMMY=1                                                           LSQ02500
C                                                                              C
      DO 361 I=2,J                                                      LSQ02510
      DUMMY=DUMMY*IV                                                    LSQ02520
      SIV(I)=SIV(I)+DUMMY                                               LSQ02530
      SDV(I)=SDV(I)+DV*DUMMY                                            LSQ02540
  361 CONTINUE                                                          LSQ02550
C                                                                              C
      SDV(1)=SDV(1)+DV                                                  LSQ02560
      SDV(11)=SDV(11)+DV*DV                                             LSQ02570
C                                                                              C
      DO 362 I=J1,J2                                                    LSQ02580
      DUMMY=DUMMY*IV                                                    LSQ02590
      SIV(I)=SIV(I)+DUMMY                                               LSQ02600
  362 CONTINUE                                                          LSQ02610
C                                                                              C
      GO TO 330                                                         LSQ02620
  370 REWIND 7                                                          LSQ02630
      REWIND 8                                                          LSQ02640
      IF(NPTS .LE. J) GO TO 900                                         LSQ02650
      SIV(1)=NPTS                                                       LSQ02660
      M=1                                                               LSQ02670
      N=0                                                               LSQ02680
      IF(.NOT. ZERO) GO TO 371                                          LSQ02690
      M=0                                                               LSQ02700
      N=1                                                               LSQ02710
      J=DEGREE                                                          LSQ02720
      J1=J+1 .                                                          LSQ02730
C                                                                              C
C        USE SIV AND SDV TO GENERATE THE LEAST SQUARES MATRIX.                 C
C                                                                              C
  371 DO 373 I=1,J                                                      LSQ02740
C                                                                              C
      DO 372 K=1,J                                                      LSQ02750
      MATRIX(I,K)=SIV(I-M+K)                                            LSQ02760
  372 CONTINUE                                                          LSQ02770
C                                                                              C
      MATRIX(I,J1)=SDV(1+N)                                             LSQ02780
  373 CONTINUE                                                          LSQ02790
```

```
C                                                                              C
C        CALL GAUSS TO SOLVE MATRIX AND INVERT IT OF .NOT. ZERO.               C
C                                                                              C
      CALL GAUSS(MATRIX,J,A,PIVOT,SINGULR,10,.NOT.ZERO)                 LSQ2800
C                                                                              C
      IF(.NOT. SINGULR) GO TO 380                                       LSQ2810
      WRITE (6,6014)                                                    LSQ2820
      GO TO 1                                                           LSQ2830
C                                                                              C
  380 SDDEZ=0.0                                                         LSQ2840
      IF(LIST)  WRITE(6,6015)                                           LSQ2850
      IF(.NOT. ZERO) GO TO 389                                          LSQ2860
C                                                                              C
C        IF ZERO, SHIFT RESULTS TO THE RIGHT IN A.                             C
C                                                                              C
      DO 382 I=1,J                                                      LSQ2870
      A(J+2-I)=A(J+1-I)                                                 LSQ2880
  382 CONTINUE                                                          LSQ2890
C                                                                              C
      A(1)=0.0                                                          LSQ2900
      J=DEGREE+1                                                        LSQ2910
      J1=J+1                                                            LSQ2920
C                                                                              C
C        COMPUTE AND OUTPUT THE DV ESTIMATES AND THE REGRESSION ERRORS.        C
C                                                                              C
  389 LINE=10                                                           LSQ2930
      PAGE=1                                                            LSQ2940
      DO 400 I=1,NPTS                                                   LSQ2950
      READ (8,8000) IV,DV                                               LSQ2960
      DVE=A(J)                                                          LSQ2970
C                                                                              C
      DO 390 K=1,DEGREE                                                 LSQ2980
      DVE=DVE*IV+A(J-K)                                                 LSQ2990
  390 CONTINUE                                                          LSQ3000
C                                                                              C
      ERR=DV-DVE                                                        LSQ3010
      SDDEZ=SDDEZ+ERR*ERR                                               LSQ3020
      IF(.NOT.LIST) GO TO 400                                           LSQ3030
      LINE=LINE+1                                                       LSQ3040
      IF(LINE.LE.47.OR.NPTS-I.LE.6) GO TO 391                           LSQ3050
      LINE=1                                                            LSQ3060
      PAGE=PAGE+1                                                       LSQ3070
      WRITE (6,6017) TITLE,PAGE                                         LSQ3080
      WRITE (6,6015)                                                    LSQ3090
  391 WRITE (6,6016) IV,DV,DVE,ERR                                      LSQ3100
  400 CONTINUE                                                          LSQ3110
C                                                                              C
C        COMPUTE AND OUTPUT THE REGRESSION STATISTICS.                         C
C                                                                              C
      PAGE = PAGE+1                                                     LSQ3120
      UNBIAS = FLOAT(NPTS)/(NPTS-1)                                     LSQ3130
      VARD=(SDV(11)/NPTS-(SDV(1)/NPTS)**2)*UNBIAS                       LSQ3140
      VARDDE=SDDEZ/(NPTS-J)                                             LSQ3150
      VARI=(SIV(3)/NPTS-(SIV(2)/NPTS)**2)*UNBIAS                        LSQ3160
      IF(LIST) WRITE (6,6017) TITLE,PAGE                                LSQ3170
      WRITE (6,6018)                                                    LSQ3180
      DO 410 I=1,J                                                      LSQ3190
```

```
      K=I-1
      IF(.NOT.ZERO) GO TO 408
      WRITE (6,6019) K,A(I)
      GO TO 410
 408  V=MATRIX(I,I)*VARDDE
      S=SQRT(V)
      N=NPTS-J
      WRITE (6,6019) K,A(I),V,S,N
 410  CONTINUE
C
      EVEN=DEGREE.EQ. (DEGREE/2)*2
      IF((LIST.AND..NOT.(EVEN.OR.ZERO)) .OR.
     -   (EVEN.AND..NOT.(LIST.OR.ZERO)) .OR.
     -   (ZERO.AND..NOT.(LIST.OR.EVEN)) .OR.
     -   (LIST.AND.EVEN.AND.ZERO)) WRITE (6,6025)
      SDDEZ=SDDEZ/NPTS
      WRITE (6,6020)
      STDD=SQRT(VARD)
      STDE=SQRT(VARDDE)
      STDI=SQRT(VARI)
      IF(.NOT.ZERO) GO TO 413
      WRITE (6,6021) DVAR,IVAR,SDDEZ,STDE,VARDDE,STDD,VARD,STDI,VARI
      GO TO 1
C
 413  R2=1.0-((NPTS-J)*VARDDE)/((NPTS-1)*VARD)
      R=SQRT(ABS(R2))
      WRITE (6,6021) DVAR,IVAR,SDDEZ,STDE,VARDDE,STDD,VARD,STDI,VARI,
     -BLANK,R,R2
      IF(.NOT. LIST .AND. DEGREE .GT. 4) WRITE (6,6017) TITLE,PAGE
      WRITE (6,6022) DEGREE
      N=5*DEGREE-4
C
C        OUTPUT THE COVARIANCE MATRIX FROM THE REGRESSION.
C
      DO 415  I = 1,J
      WRITE (6,MFMT(N))(MATRIX(I,K),K=1,J)
 415  CONTINUE
C
      N=MIN0(NPTS-J,999)
      WRITE(6,6023) J,N
      GO TO 1
C
 900  WRITE (6,6024) J1,NPTS
      GO TO 1
C
 999  WRITE (6,6026)
      STOP
      END
```

```
      SUBROUTINE GAUSS(A,N,X,PIVOT,SINGULR,IDIM,INVERT)                 GAUS0000
C                                                                              C
C                                                                              C
      REAL A(IDIM,1),X(IDIM),MAX                                        GAUS0010
C                                                                              C
      INTEGER PIVOT(IDIM),P,PP                                          GAUS0020
C                                                                              C
      LOGICAL SINGULR,INVERT                                            GAUS0030
C                                                                              C
C  X IS THE SOLUTION VECTOR FOR B*X=Y, WHERE B IS N BY N.                      C
C  A=B:Y:I, IF INVERT = .TRUE., AND A=B:Y OTHERWISE. A IS N BY NN.             C
C  ON OUTPUT, A=B**-1:Z:C IF INVERT =.TRUE., AND A=D:Z OTHERWISE.              C
C  (HERE I IS THE IDENTITY MATRIX AND C, D, AND Z ARE ARBITRARY.)              C
C                                                                              C
C  PIVOT(J) CONTAINS THE PHYSICAL ROW INDEX OF THE JTH ROW IN A.               C
C                                                                              C
C  DEFINE THE KRONECKER DELTA AS KDELT(I,J)=(I/J)*(J/I).                       C
C                                                                              C
      KDELT(I,J)=(I/J)*(J/I)                                            GAUS0040
      SINGULR=.FALSE.                                                   GAUS0050
      N1=N+1                                                            GAUS0060
      IF(INVERT) GO TO 20                                               GAUS0070
      NN=N+1                                                            GAUS0080
      DO 10 I=1,N                                                       GAUS0090
 10   PIVOT(I)=I                                                        GAUS0100
      GO TO 100                                                         GAUS0110
C                                                                              C
C  INVERT IS .TRUE.  SET UP IDENTITY PORTION OF A.                             C
C                                                                              C
 20   NN=2*N+1                                                          GAUS0120
      DO 40  J=1,N                                                      GAUS0130
      DO 30  I=1,N                                                      GAUS0140
      A(I,J+N1)=KDELT(I,J)                                              GAUS0150
 30   CONTINUE                                                          GAUS0160
      PIVOT(J)=J                                                        GAUS0170
 40   CONTINUE                                                          GAUS0180
C                                                                              C
C  FIND OPTIMAL PIVOT ROWS AND REDUCE B TO AN UPPER TRIANGULAR MATRIX.          C
C  (THE ENTIRE A MATRIX IS ALTERED IN THIS PROCESS.)                           C
C                                                                              C
 100  NM=N-1                                                            GAUS0190
      DO 120 I = 1,NM                                                   GAUS0200
      MAX = 0.0                                                         GAUS0210
      DO 110 J = I,N                                                    GAUS0220
      JJ = PIVOT(J)                                                     GAUS0230
      SUM = 0.0                                                         GAUS0240
      DO 105 K = I,N                                                    GAUS0250
      SUM = SUM + ABS(A(JJ,K))                                          GAUS0260
 105  CONTINUE                                                          GAUS0270
C                                                                              C
      IF (SUM .EQ. 0.0) GO TO 900                                       GAUS0280
      SUM = ABS(A(JJ,I))/SUM                                            GAUS0290
      IF (SUM .LE. MAX) GO TO 110                                       GAUS0300
      P  = J                                                            GAUS0310
      PP = JJ                                                           GAUS0320
      MAX = SUM                                                         GAUS0330
 110  CONTINUE                                                          GAUS0340
```


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```
C                                                                       C
      IF(MAX.EQ.0.0) GO TO 900                                         GAUS0350
      K = PIVOT(I)                                                     GAUS0360
      PIVOT(I) = PP                                                    GAUS0370
      PIVOT(P) = K                                                     GAUS0380
      APPI = A(PP,I)                                                   GAUS0390
      II = I + 1                                                       GAUS0400
      DO 119 J = II,N                                                  GAUS0410
      JJ = PIVOT(J)                                                    GAUS0420
      R = A(JJ,I)/APPI                                                 GAUS0430
      DO 115 K = I,NN                                                  GAUS0440
      A(JJ,K) = A(JJ,K) - R* A(PP,K)                                   GAUS0450
  115 CONTINUE                                                         GAUS0460
  119 CONTINUE                                                         GAUS0470
  120 CONTINUE                                                         GAUS0480
C                                                                       C
C  SOLVE FOR X FROM THE UPPER TRIANGULAR MATRIX B.                      C
C                                                                       C
      JJ = PIVOT(N)                                                    GAUS0490
      IF (A(JJ,N) .EQ. 0.0) GO TO 900                                  GAUS0500
      X(N) = A(JJ,N1) / A(JJ,N)                                        GAUS0510
      IF(N.GE.2) GO TO 134                                             GAUS0520
      A(1,1)=1.0/A(1,1)                                                GAUS0530
      RETURN                                                           GAUS0540
C                                                                       C
  134 DO 140 I=2,N                                                     GAUS0550
      J = N + 1 - I                                                    GAUS0560
      JJ = PIVOT(J)                                                    GAUS0570
      SUM = 0.0                                                        GAUS0580
      P = J + 1                                                        GAUS0590
      DO 135 K = P,N                                                   GAUS0600
      SUM = SUM + A(JJ,K) * X(K)                                       GAUS0610
  135 CONTINUE                                                         GAUS0620
C                                                                       C
      X(J) = (A(JJ,N1)-SUM)/A(JJ,J)                                    GAUS0630
  140 CONTINUE                                                         GAUS0640
C                                                                       C
      IF(.NOT.INVERT) RETURN                                           GAUS0650
C                                                                       C
C  DETERMINE B INVERSE.                                                 C
C                                                                       C
      N2=N+2                                                           GAUS0660
      K=N-1                                                            GAUS0670
      DO 150 J=1,K                                                     GAUS0680
      JJ=N1-J                                                          GAUS0690
      I=PIVOT(JJ)                                                      GAUS0700
      AIJJ=A(I,JJ)                                                     GAUS0710
      DO 142  L=N2,NN                                                  GAUS0720
      A(I,L)=A(I,L)/AIJJ                                               GAUS0730
  142 CONTINUE                                                         GAUS0740
C                                                                       C
      JJ1=JJ-1                                                         GAUS0750
      DO 146  M=1,JJ1                                                  GAUS0760
      MM=PIVOT(M)                                                      GAUS0770
      AMMJJ=A(MM,JJ)                                                   GAUS0780
      DO 144  L=N2,NN                                                  GAUS0790
      A(MM,L)=A(MM,L  )-AMMJJ*A(I,L)                                   GAUS0800
```

```
  144   CONTINUE                                                    GAUS0810
  146   CONTINUE                                                    GAUS0820
  150   CONTINUE                                                    GAUS0830
C                                                                          C
        I=PIVOT(1)                                                  GAUS0840
        AI1=A(I,1)                                                  GAUS0850
        DO 160  L=N2,NN                                             GAUS0860
        A(I,L)=A(I,L)/AI1                                           GAUS0870
  160   CONTINUE                                                    GAUS0880
C                                                                          C
        DO 170  J=1,N                                               GAUS0890
        I=PIVOT(J)                                                  GAUS0900
        DO 165  L=1,N                                               GAUS0910
        A(J,L)=A(I,L+N1)                                            GAUS0920
  165   CONTINUE                                                    GAUS0930
  170   CONTINUE                                                    GAUS0940
C                                                                          C
        RETURN                                                      GAUS0950
  900   SINGULR=.TRUE.                                              GAUS0960
        RETURN                                                      GAUS0970
        END                                                         GAUS0980
```

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