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DETERMINING ELASTIC MODULI OF MATERIALS IN PAVEMENT SYSTEMS BY SURFACE DEFLECTION DATA, A FEASIBILITY STUDY

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ABSTRACT

The determination of the elastic, or Young's, modulus, E, of the materials in each layer in an n-layered pavement system given the number, order, thicknesses, and Poisson's ratios of the layers, and the surface load and deflection data, is not possible using the classical theory of elasticity alone. This report develops some assumptions and techniques, based on the effective modulus concept, Burmister's deflection equation, the finite element method, and the concepts of beams and plates on elastic foundations, which yield mathematical solutions for such moduli.

INTRODUCTION

The determination of the elastic, or Young's, modulus, E, of the materials in each layer in an n-layered pavement system is desirable for --

1. determining deterioration in pavement systems as reflected in changes in moduli, and hence the need for rehabilitation;

2. determining the structural behavior of pavement materials and pavement systems for the purpose of optimizing pavement designs; and

3. establishing quality control techniques during construction.

A preliminary investigation of n-layered pavement systems by the authors has shown that given the number, order, thicknesses, and Poisson's ratios of the layers, and the surface load and the dynaflect deflection data it is not possible to utilize the classical theory of elasticity alone to determine the elastic moduli of the materials in each layer. Therefore other methods must be employed to determine the elastic moduli of the materials in multi-layer systems.

OBJECTIVE

The objective of this research was to investigate the possibility of determining the elastic moduli of the materials in multi-layer pavement systems from dynaflect deflection data.

SCOPE

The following concepts and procedures were investigated as to their individual and combined potentials:

1. the effective moduli of pavement systems,

2. Burmister's equation,

3. the finite element method, and

4. the concepts of beams and plates on elastic foundations.

EFFECTIVE MODULUS OF A PAVEMENT SYSTEM

The concept of an effective modulus of a pavement system is based on a spring analogy extended to columns and on Boussinesq's settlement equation.

Spring Analogy

Consider a simple two-layer pavement system. If it is assumed that μ, Poisson's ratio, is zero for each layer, and that both layers are of finite depth, the pavement system reduces to a spring system composed of a connected column of two subsprings (layers in the original problem), which may be analyzed as noted in reference 1.

Given the system in Figure 1, one may write

$$X_1 = k_\alpha \chi \delta_1 - k_\alpha \chi \delta_2, \tag{1}$$

$$X_2 = -k_\alpha \chi \delta_1 + (k_\alpha + k_\beta) \chi \delta_2, \tag{2}$$

$$\delta_1 = \delta_\alpha + \delta_\beta, \tag{3}$$

$$\delta_2 = \delta_\beta, \text{ and} \tag{4}$$

$$X_2 = 0 \text{ (no external force)}, \tag{5}$$

where

δ_1 and δ_2 are the deflections at the upper boundaries of layers 1 and 2, respectively,

δ_α and δ_β are the deflections within the first and second layers, respectively,

X_1 and X_2 are the external loads applied to the upper boundaries of layers 1 and 2, respectively, and

k_α and k_β are the spring constants of the first and second layers, respectively.

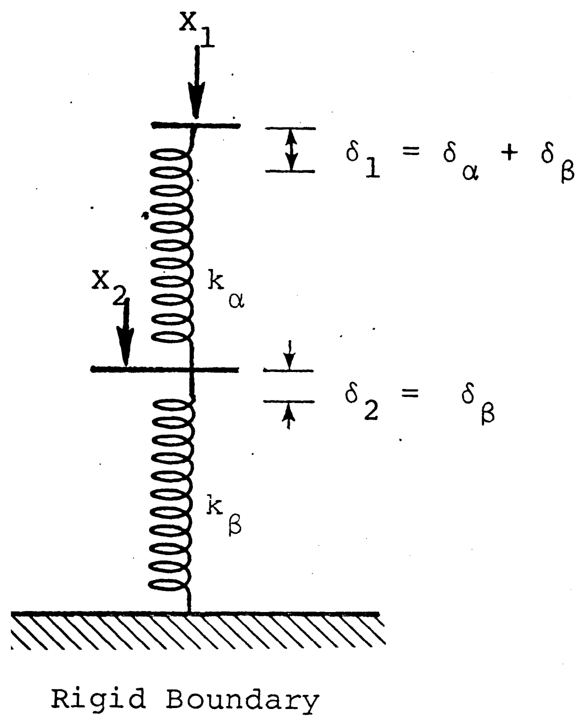

Figure 1. A two-layer spring system.

In the two-layer spring system, if the external loac and X_2 and the stiffnesses k_α and k_β are known, the two unknown deflections, δ_1 and δ_2, can be determined using equations 1 and 2.

In the inverse problem, only X_1 and δ_1 are given. Rewriting equations 1 through 5, one obtains

$$k_\alpha = \frac{X_1}{\delta_1 - \delta_2}, \text{ and} \tag{6}$$

$$k_\beta = \frac{X_1}{\delta_2}. \tag{7}$$

Therefore, the solution for k_α and k_β involves three unknowns, k_α, k_β, and δ_2, in only two equations, equations 6 and 7. Thus, there are an infinity of solutions of the form

$$(k_\alpha, \frac{k_\alpha \chi X_1}{(k_\alpha \chi \delta_1) - X_1})$$

However, there is one other experimentally measurable parameter, k_{eff}, which is the effective stiffness of the system. This parameter is defined by

$$X_1 = k_{eff} \chi \delta_1, \tag{8}$$

which implies that

$$\frac{1}{k_{eff}} = \sum \frac{1}{k_i}. \tag{9}$$

Intuitively, this concept appears to give one additional equation which may be used in conjunction with equations 6 and 7 to fully determine k_α and k_β. However, equation 9 may be derived from equations 1 and 2, by rewriting them as

$$\delta_2 = \frac{k_\alpha \chi \delta_1}{k_\alpha + k_\beta} \tag{10}$$

and

$$X_1 = k_\alpha \chi \delta_1 - \frac{k_\alpha^2 \chi \delta_1}{k_\alpha + k_\beta} = \frac{k_\alpha \chi k_\beta}{k_\alpha + k_\beta} \chi \delta_1 = k_{eff} \chi \delta_1. \tag{11}$$

Therefore, equation 9 does not increase the row dimension of the coefficient matrix.

Extension of Spring Analogy to Columns

As mentioned in the previous section, k_{eff} for a spring system is an experimentally measurable quantity. To extend the concept of k_{eff} to a three-dimensional problem, one needs to determine the equivalent of k in the layered system. Consider a column of height h, cross sectional area A, and modulus E, for such a column under a compressive force P, the deflection at the top is

$$\delta = \frac{P \chi h}{A \chi E}, \tag{12}$$

or

$$P = \frac{A \chi E}{h} \chi \delta, \tag{13}$$

which is reminiscent of the spring relation

$$P = k \chi \delta. \tag{14}$$

Thus, one can see that the form $\frac{AE}{h}$ is the "stiffness" of a column. Extending this reasoning to an n-layered system, one may write

$$\frac{1}{(E/h)_{eff}} = \sum \frac{1}{(E/h)_i}, \tag{15}$$

or

$$\left(\frac{h}{E}\right)_{eff} = \sum \left(\frac{h}{E}\right)_i, \tag{16}$$

where

h_i is the thickness of the i^{th} layer, and $h_{eff} = \sum h_i$.

The validity of the above approach must be demonstrated using either known data or the Chevron[2,3] technique in combination with Boussinesq's Settlement Equation (which is described below).

Boussinesq's Settlement Equation

Boussinesq's settlement equation[4] for the deflection under a flexible plate is

$$\delta = \frac{2 \chi (1 - \mu^2)}{E} \chi p \chi r, \tag{17}$$

where

p is the load intensity, and r is the radius of the bearing area.

Thus, one can see that treating an n-layered system as a one-layered system, under the assumption that nonhomogeniety dies not radically affect equation 17, will yield

$$E_{eff} = \frac{2 \chi (1 - \mu^2) \chi p \chi r}{\delta}, \tag{18}$$

where

E_{eff} is the effective modulus of the entire system.

E_{eff} has been empirically related to the E_i's of the layers as

$$E_{eff} = \frac{\sum (h_i \chi E_i)}{\sum h_i} \tag{19}$$

by Vaswani[5]. However, equation 16 is a potentially more rewarding relationship between E_{eff} and the E_i's.

BURMISTER'S DEFLECTION EQUATION

Burmister's equation (an extension of Boussinesq's settlement equation) for deflections under a flexible bearing area for a two-layer elastic system[4] is

$$\delta = \frac{2 \chi (1 - \mu^2)}{E_2} \chi p \chi r \chi F_w \qquad (20)$$

where

p is the load intensity,

r is the radius of the bearing area, and

F_w, the settlement coefficient, is a function of r/h_i and E_1/E_2 (charts for F_w are given in reference 4).

This equation (when the dynaflect data are known) yields a solution for E, when E_2 is known and solutions for E_1 and E_2 when E_1/E_2 is known.

FINITE ELEMENT METHOD

The finite element method can yield a complete solution for the n E_i's and n δ_i's in an n-layered pavement system, if, in addition to the number, order, thicknesses and Poisson's ratios of the layers, and the external load, n of the $2 \chi n$ E_i's and δ_i's are known. (The dynaflect deflection data, of course, yields the value of δ_1. Also, Vaswani's soil classification scheme would give the design E_s, subgrade modulus[6]). However, this solution becomes progressively more difficult to achieve as the number of unknown E_i's increases. Thus knowing n-1 of the E_i's and 1 of the δ_i's the solution is much simpler than that when, say, n-3 of the E_i's and 3 of the δ_i's are known. Furthermore, these E_i's and δ_i's are not directly available for analysis. Thus, auxiliary methods must be employed to obtain them.

BEAMS AND PLATES ON ELASTIC FOUNDATIONS

Given a two-layer system composed of an infinitely long beam supported on an elastic foundation (spring foundation), and a point load, the theory of beams on elastic foundations[7,8] states:

$$y_x = \frac{P \chi \beta}{2 \chi k} \chi e^{-\beta x} \chi (\cos\beta x + \sin\beta x) \qquad (21)$$

and

$$\theta_x = - \frac{P \chi \beta^2}{k} \chi e^{-\beta x} \sin\beta x \qquad (22)$$

where

y_x is the deflection at point x (x = o directly under the load),

θ_x is the slope of the deflection curve at x,

β equals $\left(\frac{k}{4 \chi E \chi I}\right)^{1/4}$,

k is the spring modulus of the foundation,

E is the elastic modulus of the beam,

I is the moment of inertia (second moment of area) of the beam and,

P is the point load at x = o.

When values for y_x and θ_x are determined from dynaflect deflection data, equations 21 and 22 may be used to determine E and k.

The application of these results to pavement deflections (really the theory of plates on elastic foundations[9]) requires that the rigidity of a plate be used in place of the rigidity of a beam. This is accomplished by simply substituting $Eh^3/(12(1 - \mu^2)$ for EI in the expression for β.[8] Thus an approximation for pavement deflections may be obtained by using equations 21 and 22 where

$$\beta = \left(\frac{3 \chi (1 - \mu^2) \chi k}{E \chi h^3}\right)^{1/4} \qquad (23)$$

In this manner, dynaflect data may be employed to determine E of the top layer of a pavement system and the combined k of the remaining layers.

The theory of plates on elastic foundations would, of course, yield better solutions than this extension of the theory of beams on elastic foundations would yield for E and k in a two-layer system. However, equations 21, 22, and 23 have analytical solutions, whereas the equivalent system of equations for plates on elastic foundations do not. Solutions to the plate equations require iterative improvement techniques and they are solvable in only certain instances. Thus, the authors feel that equations 21, 22, and 23 constitute an acceptable engineering approximation to the problem of plates on elastic foundations.

SOLUTIONS

The three methods discussed above can be used for determining the elastic moduli of the materials in a pavement system. Based on these methods, five possible algorithms have been prepared for solution of two-layer systems, and nineteen possible combinations of algorithms and subalgorithms have been prepared for solution of three-layer systems. These algorithms are given in the Appendix.

CONCLUSIONS

E_1 and E_2 for two-layer pavement systems can be determined from various combinations of Burmister's procedure, the finite element method, and the E_{eff} concept. The requirements for solution are that either E_1 be known from the theories of beams and plates on elastic foundations or that $E_2 = E_s$ be known from Vaswani's soil classification scheme.

E_1, E_2, and E_3 for three-layer pavement systems can be determined from various combinations of Burmister's procedure, the finite element method, and E_{eff} concept, and the treatment of combinations of layers as single layers. The requirements for solution are that both E_1 and $E_3 = E_s$ be known from the theories of beams and plates on elastic foundations and Vaswani's soil classification scheme, respectively.

RECOMMENDATIONS

This report has demonstrated that two and three-layer problems are theoretically solvable. Thus, the authors recommend that the techniques presented in this report be systematically employed, evaluated, and, if necessary, modified based on field data. The authors further recommend that the most appropriate techniques as determined from such evaluations be presented to the Department in implementable forms such as computer programs or sets of graphs.

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APPENDIX

SOLUTION ALGORITHMS

Figures A-1, A-2, and A-3 illustrate the notation used in the solution algorithms.

Two-Layer Systems

Algorithm 1

1. Estimate E_2 using Vaswani's soil classification scheme[5], (VSCS).
2. Determine E_1 using Burmister's equation.[4]

Algorithm 2

1. Determine E_1 using equations 21, 22, and 23.
2. Determine E_2 and δ_2 using the finite element method, (FEM).

Algorithm 3

1. Determine E_1 using equations 21, 22, and 23.
2. Determine E_{eff} using equation 18.
3. Determine E_2 using equation 16.

Algorithm 4

1. Estimate E_2 using VSCS.
2. Determine E_{eff} using equation 18.
3. Determine E_1 using equation 16.

Algorithm 5

1. Estimate E_2 using VSCS.
2. Determine E_1 and δ_2 using the FEM.

Three-Layer Systems

Algorithm 6

1. Determine E_1 using equations 21, 22, and 23.
2. Estimate E_3 using VSCS.
3. Determine E_{ff} using equation 18.
4. Determine E_2 using equation 16.

Algorithm 7

1. Determine E_1 using equations 21, 22, and 23.

2. Estimate E_3 using VSCS.

3. Determine E_2, δ_2, and δ_3 using the FEM.

Subalgorithm A

1. Given E_1, E_{23}, and E_3.

2. Determine E_2 using

$$\frac{h_{23}}{E_{23}} = \frac{h_2}{E_2} + \frac{h_3}{E_3}. \tag{A1}$$

Subalgorithm B

1. Given E_1, E_{12}, and E_3.

2. Determine E_2 using

$$\frac{h_{12}}{E_{12}} = \frac{h_1}{E_1} + \frac{h_2}{E_2}. \tag{A2}$$

Algorithms 8, 9, 10

1. Treat the top two layers as a single layer.

2. Apply Algorithm 1 to determine E_{12} and E_3.

(8) 3. Determine E_1 using equations 21, 22, and 23.

(9) 3. Determine E_1 using steps 1 and 2 of any Algorithm 17 through 20.

(10) 3. Determine E_1 using steps 1 and 2 of any Algorithm 21 through 24.

4. Apply Subalgorithm B.

Algorithms 11, 12, 13

1. Treat the top two layers as a single layer.

2. Apply Algorithm 4 to determine E_{12} and E_3.

(11) 3.
(12) 3. } Same as Algorithms 8, 9, 10, respectively.
(13) 3.

4. Apply subalgorithm B.

Algorithms 14, 15, 16

1. Treat the top two layers as a single layer.

2. Apply Algorithm 5 to determine E_{12} and E_3.

(14) 3.
(15) 3. } Same as Algorithms 8, 9, 10, respectively.
(16) 3.

4. Apply Subalgorithm B.

Algorithms 17, 18, 19, 20

1. Treat the bottom two layers as a single layer.

2. Apply Algorithm 2 to determine E_1 and E_{23}.

(17) 3. Determine E_3 using VSCS.

(18) 3. Determine E_3 using steps 1 and 2 of any Algorithm 8 through 10.

(19) 3. Determine E_3 using steps 1 and 2 of any Algorithm 11 through 13.

(20) 3. Determine E_3 using steps 1 and 2 of any Algorithm 14 through 16.

4. Apply Subalgorithm A.

Algorithms 21, 22, 23, 24

1. Treat the bottom two layers as a single layer.

2. Apply Algorithm 3 to determine E_1 and E_{23}.

(21) 3.
(22) 3.
(23) 3. } Same as Algorithms 17, 18, 19, 20, respectively.
(24) 3.

4. Apply Subalgorithm A.

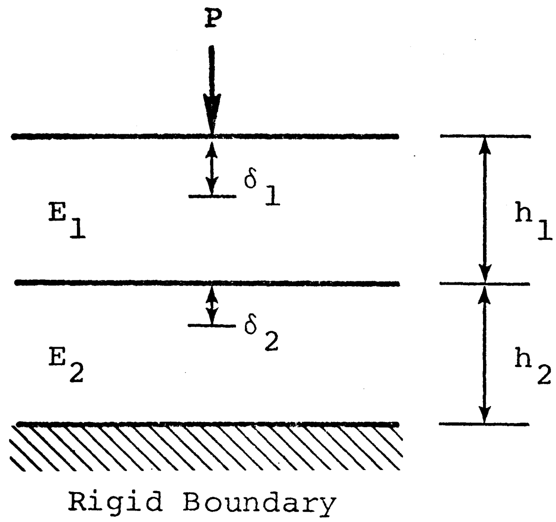

Figure A1. Two-layer system.

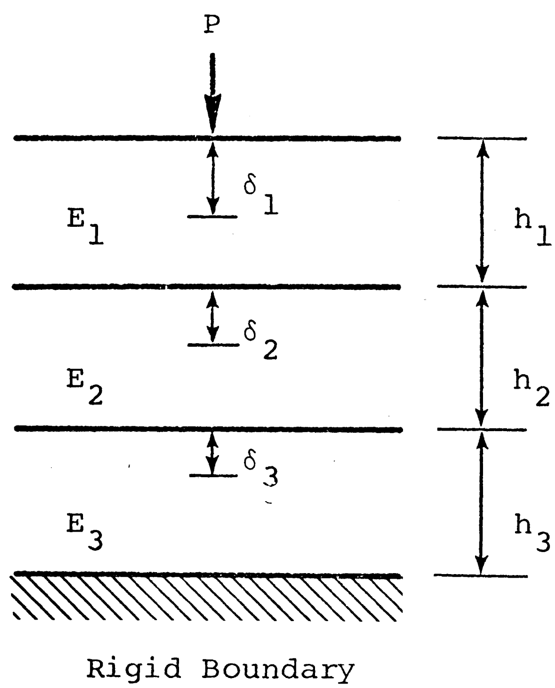

Figure A2. Three-layer system.

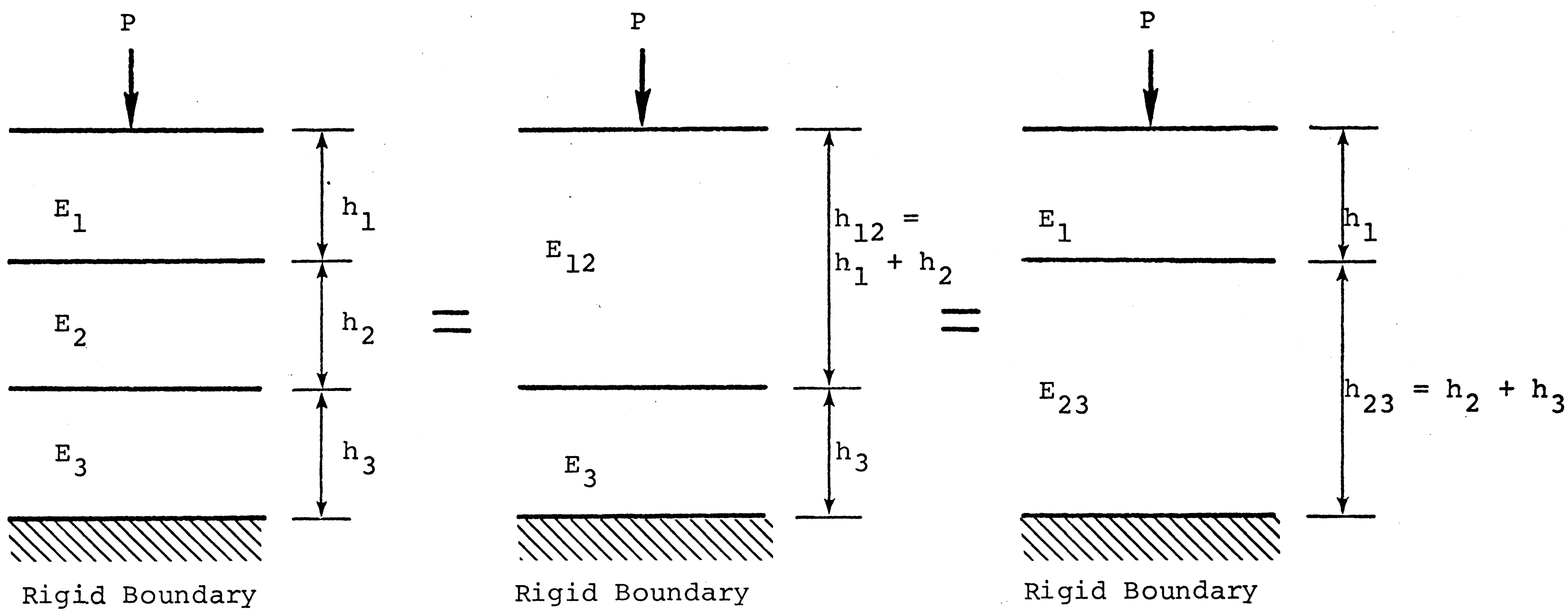

Figure A3. Two-layer representations of a three-layer system.

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