PAVEMENT DESIGN & PERFORMANCE STUDY
PHASE B: DEFLECTION STUDY

Interim Report No. 2

Subgrade Evaluation Based on Theoretical Concepts

by

N. K. Vaswani
Highway Research Engineer

Virginia Highway Research Council
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SUMMARY

Evaluations of pavement soil subgrades for the purpose of design are mostly based on empirical methods such as the CBR, California soil resistance method, etc. The need for the application of theory and the evaluation of subgrade strength in terms of the modulus of elasticity in place of empirical methods is essential for rational design techniques.

In this investigation, the maximum deflection of the pavement and the shape of the deflected basin were utilized to determine the modulus of elasticity of the subgrade. The shape of the deflected basin is defined by the word "spreadability", which is in turn defined as the average deflection as a percentage of the maximum deflection.

An evaluation chart was developed. By means of this chart the modulus of elasticity of the subgrade can be determined if the maximum and four other deflections in the deflected basin are known. The same principle could be applied to develop a similar type of evaluation chart based on the information of the deflected basin collected by any other agency.

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INTRODUCTION

The ability of a soil mass to resist displacement due an applied load is a function of the amount, duration and/or frequency of the load and the elastic properties of the soil mass. The elastic properties of the soil are theoretically defined by the modulus of elasticity and Poisson's ratio.

None of the methods adopted to date for the evaluation of subgrades (e.g., the California bearing ratio — CBR; group index — G. I.; Hveem's resistance value — R; plate bearing test; soil classification, or triaxial test) express the soil behavior in terms of the modulus of elasticity. Hence, the pavement designs so far developed by various agencies are empirical and do not involve the basic elastic properties of the materials.

Vertical displacements of the pavement surface as measured by the Benkelman beam or the dynaflect are now commonly used in evaluating the combined structural strength of the pavement layers. Both of these devices measure the maximum and other vertical displacements within the deflected basin caused by an applied load. Either device could therefore be utilized for evaluating the elastic properties of the subgrades.

PURPOSE

The purpose of the work reported here was as follows:

(1) To develop by theoretical analysis a suitable method for evaluating the subgrade support in terms of the modulus of elasticity determined from the vertical displacements measured in a deflected basin.

(2) To determine whether this theoretical analysis could be applied to the data obtained from field satellite projects.

SCOPE

The study was divided into three parts:

(1) A theoretical evaluation of the properties of the subgrade for a single layer system and a two layer system.

(2) An evaluation of field data from satellite projects by means of the theoretical correlation achieved in (1) above.

(3) The application of (1) and (2) above to design problems.

THEORETICAL EVALUATION OF THE PROPERTIES OF THE SUBGRADE

An equation for vertical displacement based on Terzaghi's equation(1) for the vertical displacement under a load on the top horizontal surface of a semi-infinite elastic solid is as follows:

$$d = \frac{P}{E_s} \quad \frac{(1 - u^2)}{f\ (r)} \tag{1}$$

where

d	=	the vertical displacement on the surface at a distance r from the applied load
P	=	applied load
E_s	=	modulus of elasticity of the semi-infinite subgrade
u	=	Poisson's ratio
f(r)	=	function of the horizontal distance from the load

The surface deflections at 0, 1', 2', 3' and 4' for a 9,000- and a 11,000-lb. wheel load for different moduli of elasticity and Poisson's ratios of the semi-infinite layer were calculated by means of a computer program(2) based on elastic theory and are given in Table I. This table and equation (1) show that (a) as d increases u decreases and vice versa; and (b) as d increases E_s decreases, and vice versa. In this investigation u was kept constant and equal to 0.47 for the following reasons: (a) Deflection

TABLE 1

CALCULATION OF THE SPREADABILITY ON A SEMI-INFINITE SINGLE LAYER

Wheel Load* (lb.)	Subgrade Modulus	Poisson's Ratio	Distance from load center					Spreadability, (percent)
			0	1'	2'	3'	4'	
			Deflections in inches					
			d_{max}	d_1	d_2	d_3	d_4	
11,000	1,000	0.5	0.7426	$.309\,d_{max}$	$.149\,d_{max}$	$.099\,d_{max}$	$.074\,d_{max}$	32.62
11,000	1,000	0.4	0.8317	$.309\,d_{max}$	$.149\,d_{max}$	$.099\,d_{max}$	$.074\,d_{max}$	32.62
11,000	1,000	0.3	0.9010	$.309\,d_{max}$	$.149\,d_{max}$	$.099\,d_{max}$	$.074\,d_{max}$	32.62
11,000	3,000,000	0.5	2.475×10^{-4}	$.309\,d_{max}$	$.149\,d_{max}$	$.099\,d_{max}$	$.074\,d_{max}$	32.62
9,000	100,000	0.5	6.717×10^{-3}	$.277\,d_{max}$	$.134\,d_{max}$	$.089\,d_{max}$	$.067\,d_{max}$	31.35
9,000	100,000	0.4	7.523×10^{-3}	$.277\,d_{max}$	$.134\,d_{max}$	$.089\,d_{max}$	$.067\,d_{max}$	31.35
9,000	100,000	0.3	8.150×10^{-3}	$.277\,d_{max}$	$.134\,d_{max}$	$.089\,d_{max}$	$.067\,d_{max}$	31.35
9,000	340,000	0.47	2.052×10^{-3}	$.277\,d_{max}$	$.134\,d_{max}$	$.089\,d_{max}$	$.067\,d_{max}$	31.35
9,000	3,000,000	0.15	2.918×10^{-4}	$.277\,d_{max}$	$.134\,d_{max}$	$.089\,d_{max}$	$.067\,d_{max}$	31.35

* Tire pressure = 70 psi over a circular area of 6.4 inch radius.

and spreadability are much less if dependent upon u than upon E; and (b) any values of deflection and spreadability obtained for varied values of u and E could be obtained by keeping u constant and varying E. Based on these two assumptions the relationship between the maximum deflection under the load (d_{max}) and E_s in equation (1) was calculated and found to be as follows:

$$E_s \, d_{max} = 700 \text{ lb./in.} \qquad (2)$$

To determine the shape of the deflected basin, certain methods were tried to obtain a suitable evaluation of the basin such that the shape could be correlated with the maximum deflection, i. e. d_{max}. The most suitable method was found to be one based on a spreadability concept.

Spreadability could be defined as the average deflection expressed as a percentage of the maximum deflection, and in this investigation it was evaluated by the equation

$$\text{Spreadability} = S = \frac{d_{max} + d_1 + d_2 + d_3 + d_4}{5\, d_{max}} \times 100 \text{ percent},$$

where d_{max}, d_1, d_2, d_3 and d_4 are the deflections at 0, 1', 2', 3' and 4' from the center of the applied load, as shown in Figure 1. Thus, the spreadability shows the

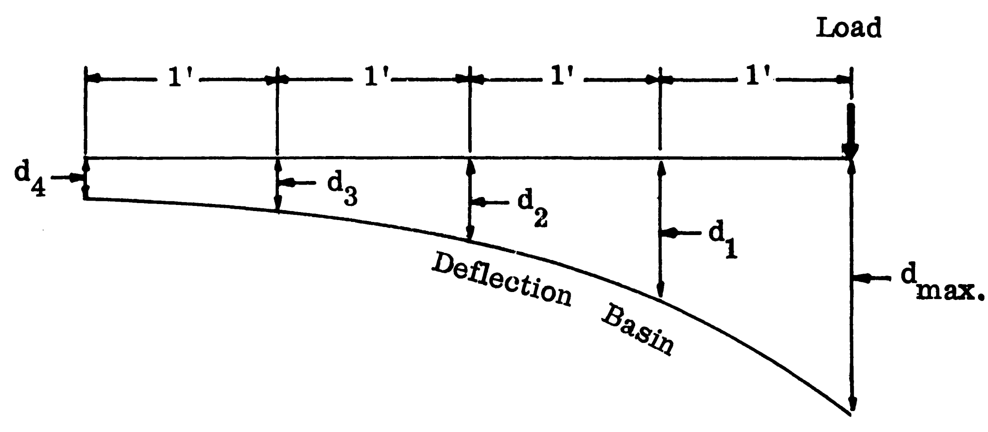


Figure 1. Evaluation of spreadability of a deflected basin.

degree to which the deformation spreads over the pavement surface. The higher the spreadability, the greater will be the extent of the deformation.

The spreadability as defined above was calculated for the theoretical deflection basins of a semi-infinite layer and varying moduli of elasticity, and as shown in Table 1 was found to be constant for a given load. It was calculated not only be means of five ordinates in the deflected basin but by various numbers of ordinates, and was found to be constant for a given number of ordinates. The spreadability as defined above — by five ordinates — was found to be 31.35 for a 9,000-lb. wheel load and 32.62 for a 11,000-lb. wheel load, as shown in Table 1. Thus, the theoretical spreadability seems to be constant for a given wheel load and any modulus of elasticity of a semi-infinite single layer.

If the properties of a subgrade were defined by maximum deflection and spreadability values, the curve of a semi-infinite single layer system for all moduli of elasticity and Poisson's ratios would be a straight line parallel to the maximum deflection scale as shown in Figure 2. The spreadability value would be a function of the load data. Thus a curve for a 9,000-lb. wheel load is shown in Figure 2 and is termed a "base line". This curve has a constant spreadability value of 31.35.

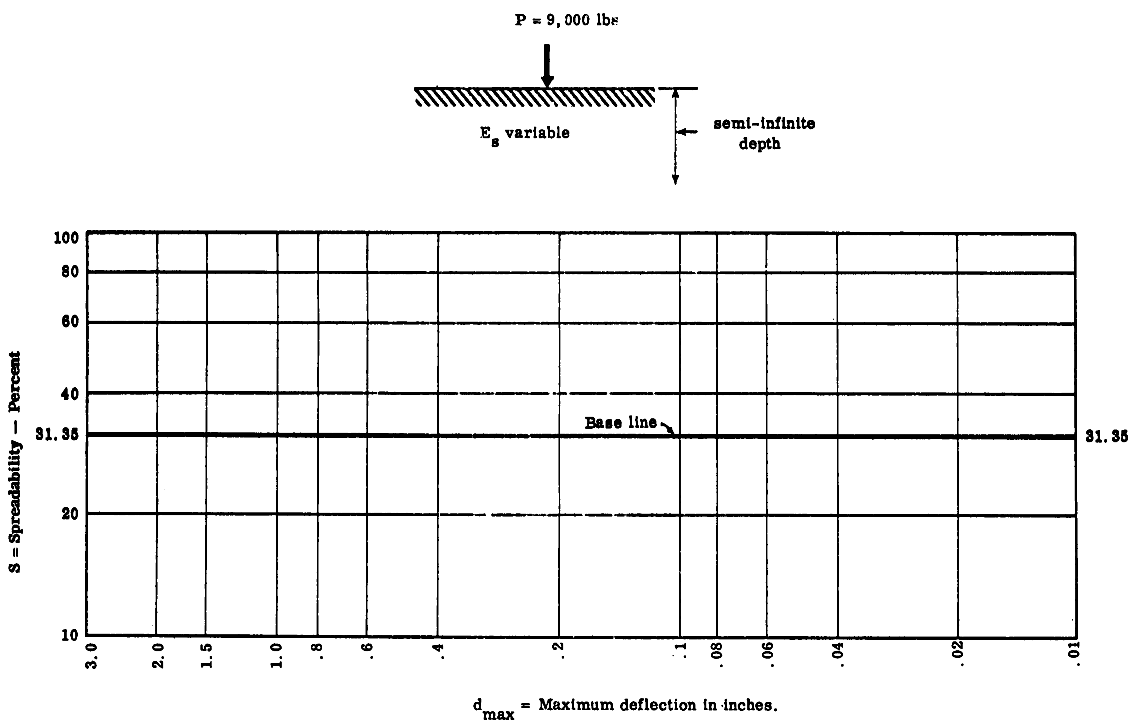


Figure 2. Spreadability value for a single layer of semi-infinite depth.

THEORETICAL EVALUATION OF THE PROPERTIES OF A TWO LAYER SYSTEM

In the theoretical analysis of the subgrade just discussed, the spreadability was found to be constant at 31.35 for a 9,000-lb. wheel load.

Field measurements of subgrade deflections have shown that the spreadability value of the subgrade varies and is usually greater than 31.35; but in very poor soils the value is less than 31.35. Since the spreadability values of the subgrade are not constant as defined by the single layer theory, it is necessary that the subgrade be considered a combination of two or more layers with their combined strengths being defined not only by the maximum deflection but also by spreadability. This consideration will then cover all the possible combinations of the maximum deflection and spreadability values obtained as shown below.

The behavior of a two layer system depends on the ratio of the modulus of elasticity of the top finite layer to the modulus of elasticity of the bottom semi-infinite layer. This ratio could be broadly classified into two types as follows:

(1) A stronger layer over a weaker layer, i.e., the ratio $\frac{E_1}{E_2}$ is greater than 1,

(2) A weaker layer over a stronger layer, i.e., the ratio $\frac{E_1}{E_2}$ is less than 1.

Each of these are discussed below. When the ratio is 1, the two layer system is equal to a single layer system of semi-infinite depth, whose properties have already been discussed.

A Stronger Layer over a Weaker Layer

When the spreadability value of the subgrade is found to be greater than for the single layer system, i.e. 31.35, the subgrade would be considered equivalent to a system of two layers with a stronger layer over a weaker layer.

The theoretical maximum deflection and spreadability values for various combinations of the moduli of elasticity of the top layer and the bottom layer were determined by elastic layered system theory.[2] The bottom layer was always considered as semi-infinite in depth while the thickness of the top layer was varied. The moduli of elasticity of the top and bottom layers were also varied, and their maximum deflection and spreadability values determined. These values are shown in Figure 3.

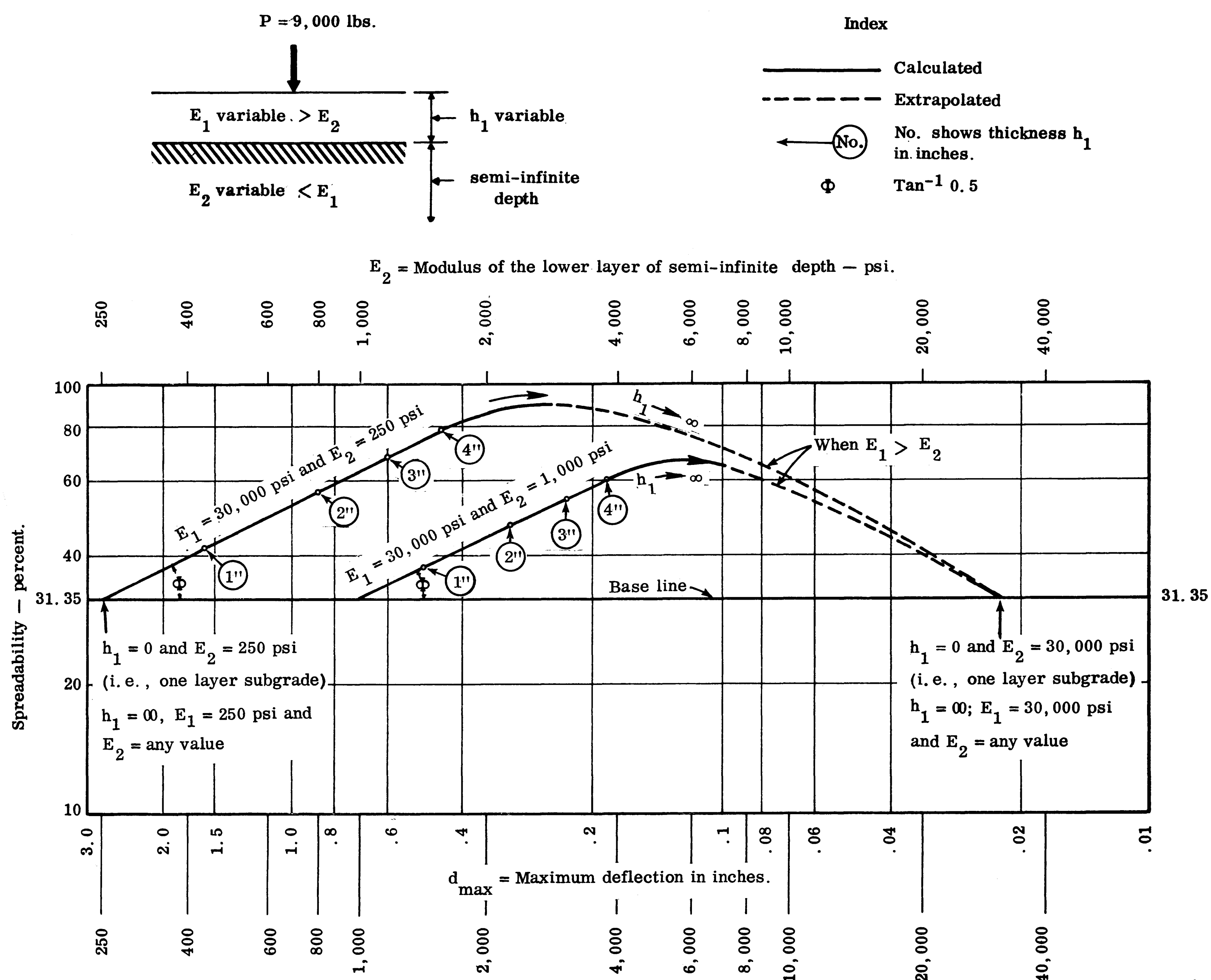


Figure 3. Two layer system in subgrade with $E_1 > E_2$.

Evaluations were carried out for different values of E_1, E_2, and h_1 to determine the relationship between the spreadability and the maximum deflection. It was found that the curve of spreadability versus maximum deflection was a straight line up to a certain depth of the material in the top layer of the subgrade. Two examples of these straight-line curves for $E_1 = 3,000$ psi and h_1 varying from 0 to 4 inches and $E_2 = 250$ and 1,000 psi respectively are shown in Figure 3. Thus up to a certain value of spreadability the relationship between spreadability and maximum deflection could be represented by the general equation

$$\log S = a \log d_{max} + b \tag{3}$$

where a and b are constants.

The value of the constant a was found to be 0.5. Thus we have the following equation:

$$\log S = 0.5 \log d_{max} + b \tag{4}$$

Thus the curves of spreadability versus maximum deflection have a slope = 0.5 with the maximum deflection scale along the horizontal axis.

As shown in equation (2) the maximum deflection of a semi-infinite subgrade is directly related to the modulus of elasticity of the subgrade. Hence in Figure 3 an additional scale of the modulus of elasticity of the subgrade could be drawn parallel to the maximum deflection scale as in Figures 3 and 4.

The curves above the base line in Figure 3 show that:

(1) As the thickness h_1 of the top stronger layer increases from zero the spreadability value increases to a certain maximum and then decreases.

(2) The modulus of elasticity of the subgrade (i.e., the two layer system with E_1/E_2 greater than 1) increases and approaches the modulus of elasticity of the top layer.

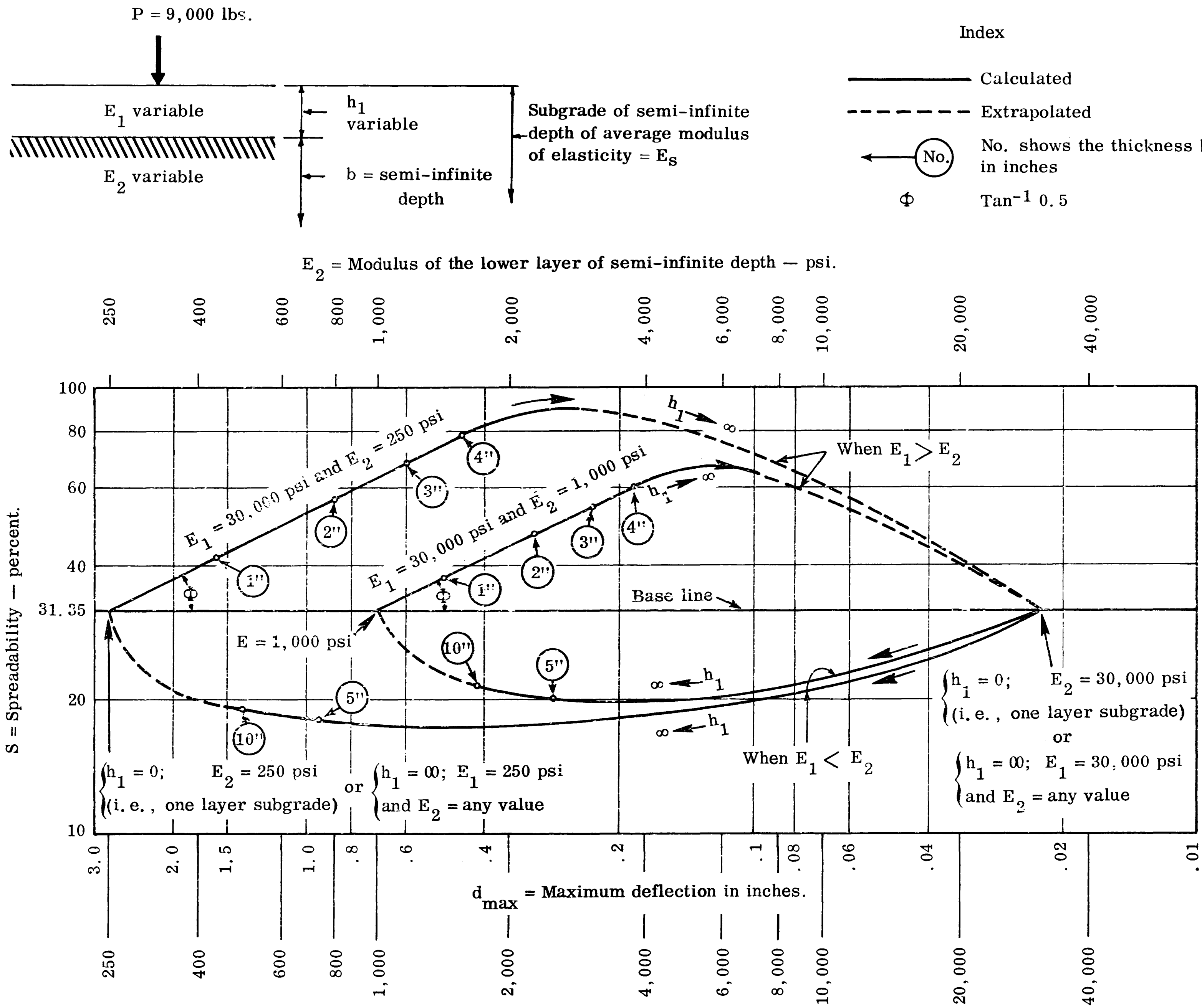


Figure 4. Two layer system in the subgrade with $E_1 \lesseqgtr E_2$.

A Weaker Layer over a Stronger Layer

When the spreadability value of the subgrade is found to be less than for the single layer system, i. e. 31. 35, the subgrade could be considered as equivalent to a two layer system with a weaker layer over a stronger layer.

The maximum deflection and spreadability values for two combinations of moduli of elasticity of the top layer and the bottom layer were determined by the elastic layered theory. The bottom layer was considered as semi-infinite in depth while the thickness of the top layer was varied. The graphs of maximum deflection versus spreadability for these two combinations are shown below the base line in Figure 4. The two curves below the base line show that:

(1) As the thickness of the top weaker layer increases from zero, the spreadability value decreases to a certain minimum and then increases as shown by the two curves.

(2) The average modulus of elasticity of the subgrade, i. e., the two layer system with E_1/E_2 less than 1, decreases as the thickness of the top layer increases and approaches the modulus of elasticity of the top layer.

In Figure 5, a general soil classification for evaluating the subgrade is shown. Figure 5 is therefore treated as a general evaluation chart in this investigation, and will cover all the types of soils in Virginia.

The writer, in some of his model studies, has shown the change in the pattern of the distribution of stresses[3] in a two layer system (a) with a stronger layer over a weaker layer and (b) with a weaker layer over a stronger layer. This pattern is shown in Figure 6.

Figure 6(a) shows that the fan type distribution takes place when a layer of a higher modulus of elasticity lies over a layer of a lower modulus of elasticity; in other words, whenthe lower layer encourages bending of the bottom side of the top layer, the stress distribution will be a fan type and the failure of the top layer could only be associated with shear plane failure.

Figure 6(b) shows that when the layer underlying the top layer has a higher modulus of elasticity than the top layer a bulb type distribution takes place. In other words, when the lower layer prevents bending of the bottom side of the top layer, the stress distribution will be of a bulb type and Boussinesg's theory, or theories based on Boussinesg's evaluation, could be applied.

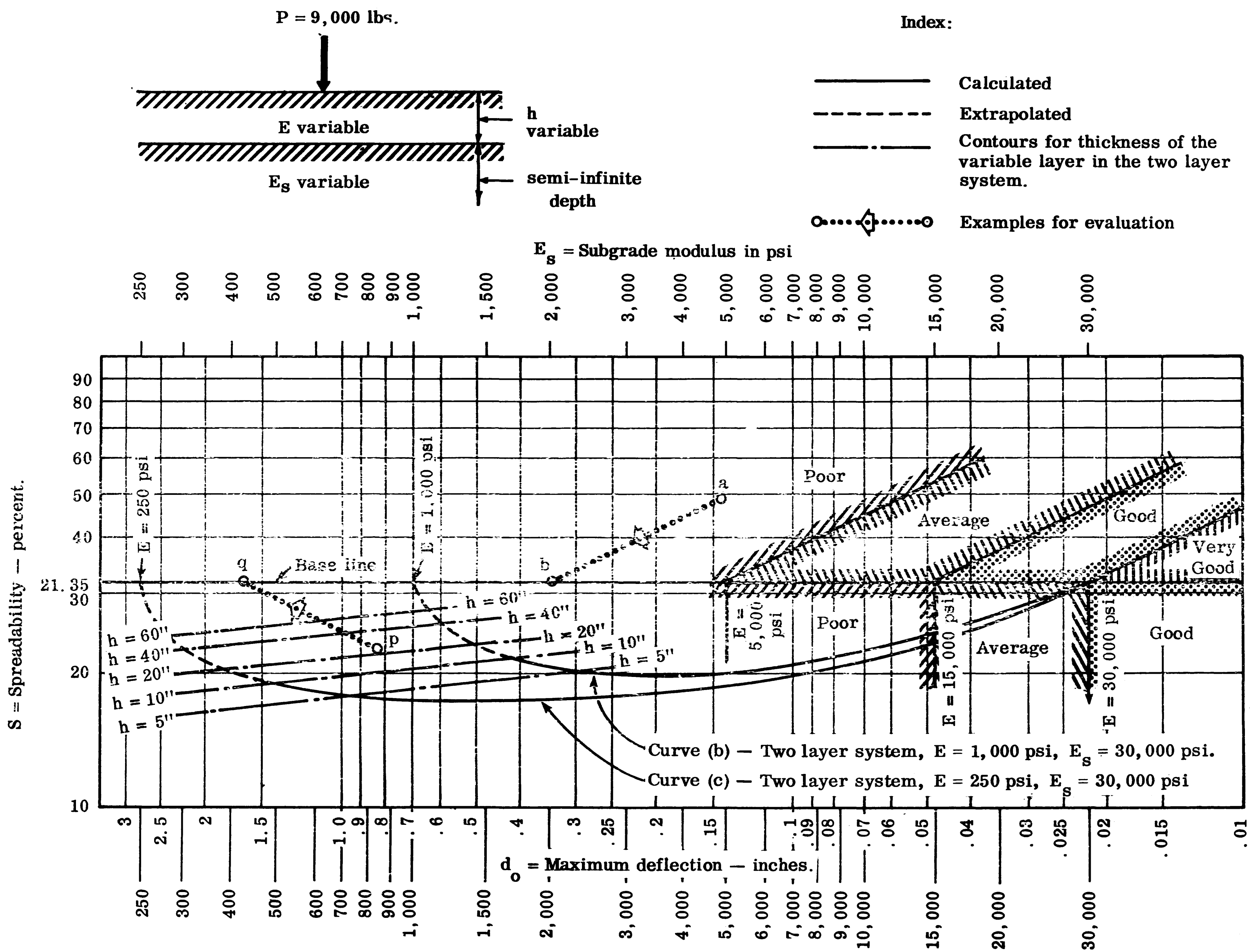
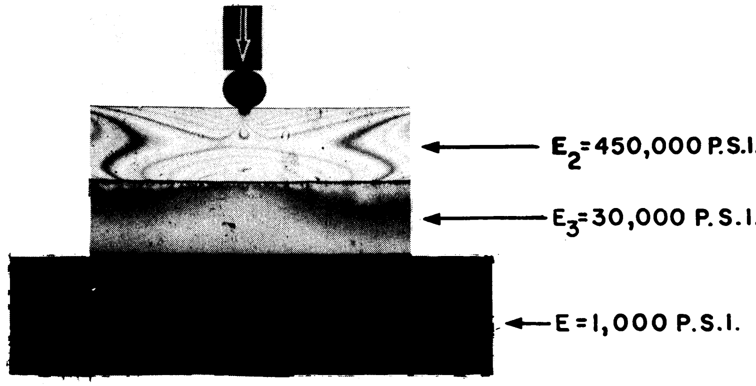
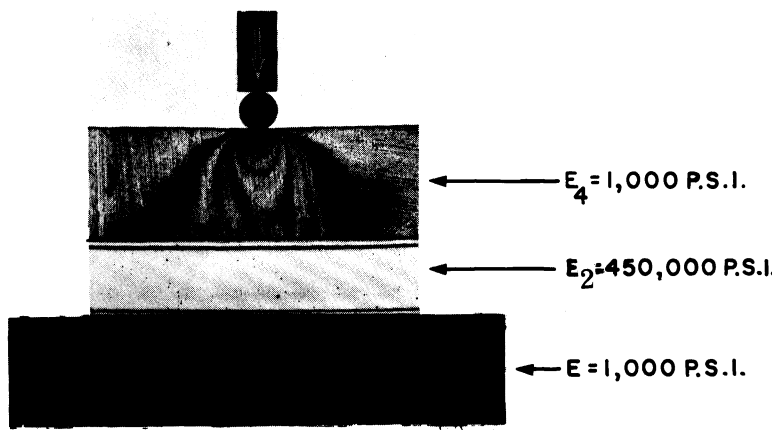


Figure 5. General evaluation chart for subgrades.



(a)



(b)

Figure 6(a). Stress distribution when a stronger layer lies over a weaker layer.

Figure 6(b). Stress distribution when a weaker layer lies over a stronger layer.

The pattern of stress distribution now could be determined by means of the spreadability factor. Thus, if the spreadability is more than a specified value (i. e., 31. 35) the distribution pattern is fan type. If the spreadability is equal to or less than the specified value (i. e., 31. 35) the distribution pattern is bulb type.

It is therefore evident that the type of design of the pavement should depend on an evalution by spreadability as well as by maximum deflection.

EVALUATION OF FIELD DATA

Two devices, among others, which could be used for the theoretical analysis discussed above for subgrade evaluations are (1) the Benkelman beam, and (2) the dynaflect.

The Benkelman beam is a static method of evaluating the vertical displacements of a deflected basin. The wheel load and tire pressure used vary. In Virginia in 1966 and before, the Benkelman beam was used with a dual wheel load of 9,000 lb., and a tire pressure of 70 psi.

The dynaflect is a dynamic method of evaluating the vertical displacement of a deflected basin. It consists of two steel wheels placed at 12 inches c/c. The peak excursions of the dynamic force vary from 600 to 2,600 lb. over the two steel wheels. The frequency of the dynamic force is 8 cycles per second. The induced motion in the material underneath the wheel is sensed by means of geophones placed at point 0 (i. e., in the center of the axis of the two wheels) and at intervals along the longitudinal axis, and the vertical displacements in the deflected basin thus determined. In Virginia, vertical displacements at 0, 1', 2', 3' and 4' are measured to evaluate the shape of the basin. To convert the dynaflect deflections into Benkelman beam deflections a correlation was carried out in Virginia and the following relationship is being used: the Benkelman beam deflection = 28. 6 x the dynaflect deflection.

The following examples discuss the evaluation of the field data by means of the theoretical evaluation discussed above.

Example No. 1: Study of the effect of strengthening the top layer on maximum deflection and spreadability.

Figure 7 shows two deflection basins obtained with maximum and minimum values of deflection on a satellite project (a) on the raw subgrade immediately before the pavement was laid, and (b) 24 hours after a 9" mat of asphaltic concrete was laid over the subgrade. Note the change in maximum deflection and spreadability values.

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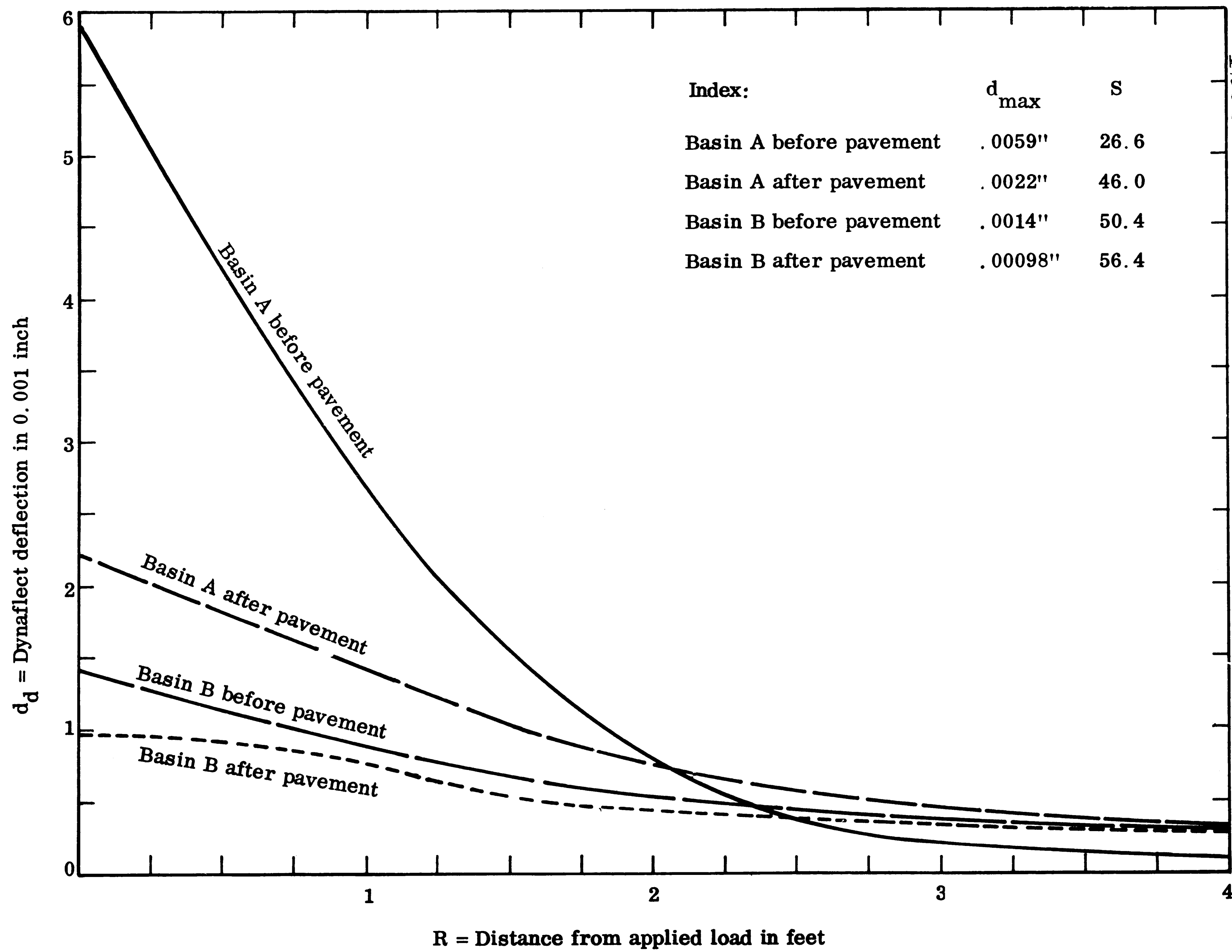

Figure 7. Typical examples of subgrade and pavement deflection basins on a field project.

Example No. 2: Study of the effect of seasonal variations on maximum deflection and spreadability.

Curve DBC in Figure 8 is an example of the highest maximum deflection values obtained in about 300 deflection sites measured on a number of satellite projects. These values were obtained on project 0064-037. In this figure, point A represents the deflection and spreadability determined on a raw subgrade on August 15. Point B represents the deflection and spreadability determined at the same location on November 1 and 2, two months after the subgrade was stabilized with soil cement. Point C represents the deflection and spreadability determined at the same location on March 20 of the following year. This example shows that two months after stabilization of the top 6" of soil the improved subgrade decreased in deflection and increased in spreadability. During the thawing period in March, with an increase in the moisture content of the subgrade, the deflection and spreadability decreased much below the value for the original raw subgrade. This tendency has been noticed on almost all projects. Table 2 shows some typical projects with maximum deflection and spreadability data during the summer before and after soil stabilization and during the thawing period.

In Figure 8 curve PQRST shows the mean of the 17 values given in Table 2 for satellite project 0064-037. Point P represents the summer evaluation of the raw subgrade, points Q and R show the increase in strength of the subgrade in the summer with an increase in strength of the cement treated subgrade. Point S illustrates the extreme effect of thawing on March 20. Point T shows that the effect of thawing had reduced by April 16.

Thus the evaluation chart in Figure 5 quantitatively shows that raw subgrades may have less deflection and sometimes more spreadability than the cement treated subgrades if the deflection data are taken on the raw subgrades when the top portion has dried.

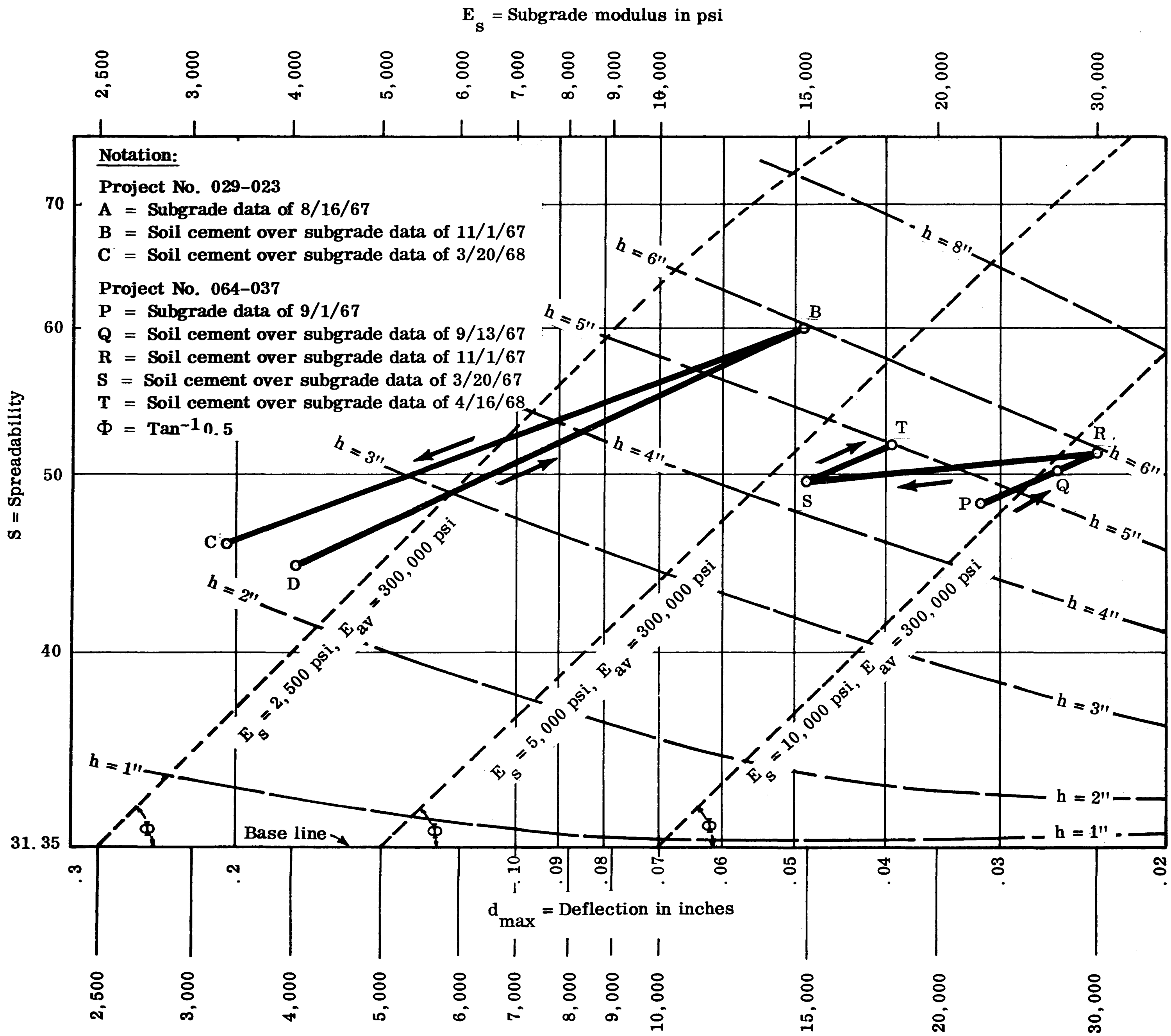

Figure 8. An example of the effect of seasonal variations on the supporting power of the subgrade.

TABLE 2

MAXIMUM DEFLECTION AND SPREADABILITY DATA SHOWING THE EFFECT OF CEMENT TREATMENT OF THE SUBGRADE AND SEASONAL CHANGES

S. No.	Project No.	Location	Top Layer	No. of Basins	d_{max} in inches		Spreadability		Date of Measurement
					Arith. Mean	Std. Dev.	Arith. Mean	Std. Dev.	
1	0029-023	WBL	Subgrade	50	.035	.017	45	7	10/31/67
2	0029-023	WBL	CTS	50	.025	.012	52	10	11/13/67
3	0064-037	WBL	Subgrade	17	.043	.036	48	8	8/16/67
4	0064-037	WBL	CTS	17	.054	.039	39	6	9/1/67
5	0064-037	WBL	CTS	17	.043	.021	43	6	9/13/67
6	0064-037	WBL	CTS	17	.045	.030	42	7	9/22/67
7	0064-037	WBL	CTS	17	.023	.013	53	6	11/1/67
8	0064-037	WBL	CTS	17	.066	.045	49	4	3/20/68
9	0064-037	EBL	Subgrade	19	.031	.028	48	8	9/1/67
10	0064-037	EBL	CTS	19	.025	.017	50	5	9/13/67
11	0064-037	EBL	CTS	19	.023	.013	52	6	11/1/67
12	0064-037	EBL	CTS	19	.048	.024	49	6	3/20/68
13	0064-037	EBL	CTS	19	.039	.012	52	5	4/16/68

APPLICATION TO FIELD PROBLEMS

The evaluation of the subgrade soil based on the theoretical evaluation has been explained in the examples given above. In this manner about 20 satellite projects were analyzed. Based on these analyses the following recommendations are made.

(1) Soil classification: Soils have been classified as poor, average, good, and very good on the basis of subgrade support strength in terms of the subgrade modulus. This classification is shown in Figure 5.

Poor soil has an E_s = 5,000 psi or less
Average soil has an E_s = 5,000 to 15,000 psi
Good soil has an E_s = 15,000 to 30,000 psi
Very good soil has an E_s greater than 30,000 psi.

(2) Subgrade evaluation for design purposes: To avoid any false effect of weather conditions on the deflection data for the subgrade, reduce the two layer subgrade system as follows:

(a) For spreadability values greater than 31.35 follow two steps. (1) Plot a point with the subgrade support value on the evaluation chart. The subgrade support value is defined by the maximum deflection and spreadability. An example of this is point a in Figure 5, which has a deflection of 0.145 inch and a spreadability of 49. (2) Draw a line through point a parallel to the inclined line, i.e., at an angle of $\mathrm{Tan}^{-1}\,0.5$ degrees with the horizontal, and let it meet the base line of spreadability = 31.35, say point b. The subgrade modulus so obtained on the base line could be assumed as the subgrade modulus for design purposes.

(b) For any project for the purpose of design, data are collected at a number of locations along the project. The average spreadability of the projects would always be greater than 31.35. Hence, it would not be necessary to design for spreadability values less than 31.35. However, some poor local conditions for spreadability values less than 31.35 might require reduction of the two layer to a one layer subgrade system. To illustrate: Draw a line through the subgrade support point on the general evaluation chart (point P in Figure 5) at an angle of $\mathrm{Tan}^{-1} 0.5$ degrees with the horizontal towards the left, as shown in Figure 5, and let it meet the base line (point Q in Figure 5). The subgrade modulus so obtained on the base line could then be assumed as the subgrade modulus.

CONCLUSIONS

1. Subgrade support strength can be defined by the modulus of elasticity of the subgrade soil instead of empirical values, and thus enable pavement designs to be made on a more rational basis.

2. The modulus of elasticity of the subgrade soil could be determined by means of dynaflect or Benkelman beam deflection devices by measuring deflections in the deflected basin.

3. Spreadability (as defined in this paper) does not change with a change in the modulus of elasticity or Poisson's ratio of a semi-infinite layer.

4. Spreadability (as defined in this paper) indicates whether the upper layer is weaker than the lower layer and vice versa.

5. The subgrade layered system should be reduced to a single layer system for better assurance of structural adequacy.

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NOTATIONS

a	=	radius of contact area, in inches
d	=	vertical displacement on the surface, in inches.
d_1, d_2, d_3 and d_4	=	vertical displacement at 1', 2', 3' and 4' from the load.
d_d	=	vertical displacement due to dynaflect in 0.001 inch.
d_{max}	=	maximum vertical displacement, in inches.
E_1	=	average modulus of elasticity of the top layer, h_1, in the subgrade, in psi.
E_2	=	average modulus of elasticity of the bottom layer of semi-infinite depth in the subgrade, in psi.
E_s	=	average modulus of elasticity of the subgrade of semi-infinite depth, in psi.
h	=	thickness of layer having $E = 300,000$ psi, in inches.
h_1	=	thickness of the top layer in the subgrade, in inches.
P	=	load, in lb.
p	=	tire pressure, in psi.
s	=	spreadability, in percent.
r	=	horizontal distance from the center of applied load, in inches.
u	=	Poisson's ratio.

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