



Development of Enhanced Pavement Deterioration Curves

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Final Report VTRC 17-R7

Standard Title Page—Report on State Project

Report No.: VTRC 17-R7	Report Date: October 2016	No. Pages: 38	Type Report: Final Contract	Project No.: 105332
			Period Covered:	Contract No.:
Title: Development of Enhanced Pavement Deterioration Curves				Key Words: pavement, service life, structural condition
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Sponsoring Agencies' Name and Address: Virginia Department of Transportation 1401 E. Broad Street Richmond, VA 23219				
Supplementary Notes:				
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FINAL REPORT

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Virginia Transportation Research Council
(A partnership of the Virginia Department of Transportation
and the University of Virginia since 1948)

Charlottesville, Virginia

October 2016
VTRC 17-R7

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Each contract report is peer reviewed and accepted for publication by staff of Virginia Transportation Research Council with expertise in related technical areas. Final editing and proofreading of the report are performed by the contractor.

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ABSTRACT

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The developed regression model predicts the average *CCI* of pavement sections for a given age and MSI value. In practice, the actual *CCI* of specific pavement sections will vary from the model-predicted condition because many (important) factors that affect deterioration are not considered in the model. Therefore an empirical Bayes (EB) method is proposed to better estimate the *CCI* of a specific pavement section. The EB method combines the recorded *CCI* of the specific section with the *CCI* predicted from the model using a weighted average that depends on the variability of individual pavement sections performance and the variability of *CCI* measurements. This approach resulted in improving the prediction of the future *CCI*, calculated using leave one out cross validation, by 21.6%.

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INTRODUCTION

One of the main goals of the Virginia Department of Transportation (VDOT) is to keep the entire road network operating at a high serviceability level. Accurate pavement performance prediction can significantly help pavement managers achieve that goal. Current pavement performance prediction models (also called deterioration models or deterioration curves) used by VDOT do not directly take into account how the pavement structural condition affects pavement performance. This will result in less than optimal decisions as research has shown that the pavement structural condition significantly affects the pavement performance (Bryce et al., 2013; Flora, 2009; and Zaghoul et al., 1998). In 2007, Stantec Consulting Services Inc. with the cooperation of H.W. Lochner Inc., henceforth referred to as Stantec, developed default pavement deterioration models for VDOT. The practice within VDOT is to use two types of performance prediction models: site specific models where sufficiently accurate site specific data are available

and default models where such accurate data are not available. Data accuracy is determined through quality checks that predict the minimum and maximum service life, which are compared with a pre-defined range of acceptance.

Although the pavement structural condition was not directly incorporated in the default deterioration models developed by Stantec, it was recognized that different pavement rehabilitation treatments have different effects on the pavement structural condition and hence future performance. Therefore, different default deterioration models were developed for the different pavement rehabilitation treatments. These rehabilitation treatments are grouped into four maintenance categories namely, preventive maintenance (PM), corrective maintenance (CM), restorative maintenance (RM), and reconstruction (RC). In summary, the approach followed by Stantec was to use the windshield pavement survey data and develop pavement performance models assuming all pavement sections had CM as a last treatment. The assumption of a last treatment was necessary as the windshield data did not record the type of the last treatment. Expert opinion from within VDOT was then combined with the developed model for CM to develop models for the remaining three maintenance categories (Stantec, 2007).

PURPOSE AND SCOPE

Purpose

The main purpose of this study was to develop a pavement deterioration regression model for bituminous sections of VDOT Interstate roads that take into account the pavement structural condition. The developed regression model can be implemented by VDOT in their pavement management system (PMS) to better account for the effect of structural condition on pavement performance.

The pavement structural condition accounts for some (significant) but not all of the variability of the performance of the pavement sections. Therefore, a methodology that combines the model predicted Critical Condition Index (*CCI*) with the measured *CCI* to account for the unexplained variability by the model is also presented and validated by (leave-one-out) cross validation. This additional step can be implemented separately from the regression.

Scope

The data used in this project were obtained from the Interstate roads in Virginia. This restriction was necessary because the primary and secondary roads in the VDOT network do not have network level structural evaluation measurements. A key aspect of this study was to adopt a model for the pavement deterioration process that can explain the observed data. The study found that a Negative Binomial model provides a good representation of the observed pavement condition (and much better representation than a model based on the normal distribution). In addition to providing a good fit to the observed data, which will result in a better regression model, the Negative Binomial model can also take into account the fact that variation in the

observed pavement condition comes from two sources. The first source is variation in performance of different pavement sections while the second source is variation due to variability in the measurement and reporting of the pavement condition.

With these two sources of variations, an empirical Bayes (EB) approach that combines the pavement performance model obtained from the Negative Binomial regression with the condition of individual pavement sections is used to estimate the performance of each pavement section. This estimate is better than what is separately achieved by either the model or the actual observations.

METHODS

There are two products from this research. The first main product is the regression model that estimates the pavement CCI based on age and MSI , which is the ratio of effective structural number (SN_{eff}) over required structural number (SN_{req}). SN_{eff} is calculated from falling weight deflectometer (FWD) data while SN_{req} is calculated based on the AASHTO 1993 design method for flexible pavements (see Bryce et al., 2013 for more details). The second product is the EB estimate of CCI that combines the model predicted CCI with the measured CCI to provide an improved estimate of the CCI .

The first product of this research, the regression model, was developed as follows:

1. From the recorded CCI , calculate the Deterioration Index (DI) as follows:

$$DI = 100 - CCI \quad (1)$$

2. Using the DI , pavement age T , and MSI , perform a negative binomial regression to determine the relationship between DI and T , and MSI . The regression provides an estimate DI_{model} of DI . The CCI_{model} can then be calculated from DI_{model} . The form is given in Equation 3.

$$DI_{model} = e^{\left(\beta_0 + \beta_1 \frac{1}{MSI^4} + \beta_2 \ln(T) + \beta_3 T\right)} = T^{\beta_2} e^{\left(\beta_0 + \beta_1 \frac{1}{MSI^4} + \beta_3 T\right)} \quad (2)$$

$$CCI_{model} = 100 - DI_{model} = 100 - e^{\left(\beta_0 + \beta_1 \frac{1}{MSI^4} + \beta_2 \ln(T) + \beta_3 T\right)} = 100 - T^{\beta_2} e^{\left(\beta_0 + \beta_1 \frac{1}{MSI^4} + \beta_3 T\right)} \quad (3)$$

The chosen model includes two terms for the pavement age T . The first term, $\ln(T)$, is included so that at $T = 0$, the resulting deterioration is zero (i.e., $DI = 0$ and $CCI = 100$). The second term, T , is included because it was found that adding T results in the model having a typical observed shape of pavement deterioration (with just $\ln(T)$ the shape of the deterioration obtained from the model is not typical of observed pavement deterioration). In the end, adding the term T was also found to be statistically justifiable as it significantly improved the model fit.

The second product of this research was to combine DI with DI_{model} to give a better estimate DI_{EB} and CCI_{EB} and determine the future pavement condition. This is performed as follows

1. DI and DI_{model} are combined to give DI_{EB} using Equation 4, which is the EB approach of combining observation with model estimate:

$$DI_{EB} = \frac{1}{\frac{\phi}{\alpha} DI_{\text{model}} + 1} DI_{\text{model}} + \left(1 - \frac{1}{\frac{\phi}{\alpha} DI_{\text{model}} + 1} \right) DI \quad (4)$$

2. CCI_{EB} , is then obtained using Equation 5:

$$CCI_{EB} = 100 - DI_{EB} \quad (5)$$

3. The estimate of the following year's condition (if no observation is available) can be obtained by estimating the pavement deterioration calculated using the developed model as shown in Equation 6:

$$CCI_{EB}^{i+1} = CCI_{EB}^i + (CCI_{\text{model}}^{i+1} - CCI_{\text{model}}^i) \quad (6)$$

In Equations 2 and 3, MSI refers to the Modified Structural Index developed by Bryce et al. (2013), T , the pavement age (since last treatment). The regression coefficients were calculated as $\beta_0, \beta_1, \beta_2$ and, β_3 . In Equation 3, ϕ is a (overdispersion) parameter also obtained from the model regression, while α is a variance correction parameter that accounts for the deviations of the pavement condition data from the theoretical modeling procedure. All these parameters were determined using the pavement condition data. An example EB calculation is in Appendix A.

This methodology was followed because it is more appropriate for the condition data of pavement sections. The reason it is more appropriate was justified as follows:

1. Use of Negative Binomial distribution and regression: the empirical distribution of DI ($100 - CCI$) values obtained from the PMS was evaluated and found to be better represented by a Negative Binomial distribution than a normal distribution.
2. Including MSI in the regression model: the Akaike Information Criterion (AIC) was used to validate the model, mainly to justify including the pavement structural condition along with pavement age as one of the parameters that determines pavement condition. Adding regression variables always improves the fit of the model. The AIC penalizes the addition of regression variables so that only statistically significant variables are included.

3. Use of the variance correction parameter α : The Negative Binomial model results from a Poisson-Gamma model where the Gamma distribution represents the variability of the deterioration of different pavement sections while the Poisson distribution represents the variability in reporting of the pavement condition (error in the measured *DI* or *CCI*). It was found that the Poisson distribution underestimates the variability in the reporting of the pavement condition, which justified the inclusion of the variance correction parameter α . However, the Poisson distribution predicts that the variance of the reporting of the pavement condition is equal to the *DI*. This implies a linear relationship with a slope of unity. Similarly, the data showed that the variance is linearly related to the *DI* but with a slope of α instead of unity.
4. Validate the use of the EB approach: the EB approach was validated using leave-one-out cross validation, which consists of leaving one observation out of the model building and then predicting the condition for the observation that was left out. This process is repeated for every observation in the data set. The model prediction capability is estimated as the mean square prediction error and compared to the mean square error prediction obtained without any modeling of the pavement deterioration process.

Step 1. Data Collection and Distribution of Pavement Condition

The MSI developed in Bryce et al. (2013) was calculated from network-level FWD data collected on the Interstate roads. The MSI data were then supplemented with the pavement *CCI*, year of condition recording, and last year of maintenance data obtained from the VDOT PMS database for the years from 2007 to 2012, 2014, and 2015. The data from the years 2007 to 2012 were those initially available when the research project started and those data were used to develop and validate the model. The data from 2014 and 2015 became available at the end of the research project and were incorporated into the final model. However, they were not used in the validation of the model, as this would have required repeating the entire analysis. The MSI incorporates the information about deflection testing, pavement thickness, and traffic. The data were aggregated using the pavement management section currently in use. From the PMS data, the age of the pavement was calculated as the difference between the year of condition reporting and the last year of recorded maintenance. The total data consisted of 3,473 observations for the years from 2007 to 2012 and 1,560 observations from the years 2014 and 2015. To evaluate the *CCI* distribution, the *DI*, which is the complement of the *CCI*, was defined as shown in Equation 1. The reason *DI* is defined is for mathematical convenience as it was found that the *DI* and not the *CCI* follows the Negative Binomial distribution. The probability density function of the Negative Binomial distribution, which is also a compound Poisson-gamma distribution, is given in Appendix B along with the Poisson and Gamma distributions.

Step 2. Development of Deterioration Model

Because the distribution of the pavement DI was found to be well represented by a Negative Binomial distribution, the default pavement deterioration model was obtained using Negative Binomial regression, which is a form of a Generalized Linear Model (GLM) regression. Other than the fact the DI values are distributed as a Negative Binomial distribution, one of the advantages of Negative Binomial regression is that its natural link function is the exponential function, which was used in 2007 by Stantec to develop the default pavement deterioration models. The final model used is given by Equation 2.

Pavement condition data consist of censored data, where censoring occurs because of treatments applied to the pavement. This can lead to a biased deterioration model. The key idea is to recognize that data at higher pavement ages are mostly those of pavement sections that performed well; pavement sections that did not perform well are generally treated before reaching an older age. As such, data at older ages do not represent the performance of all pavement sections and are therefore biased. If not accounted for, this bias will result in a biased pavement deterioration model. For this reason, the model was fitted to a range of pavement age where this biasing effect is believed to be minimal. This was determined to be for data up to and including an age of 10 years.

The model parameters were determined by maximizing the likelihood function. Because the sections have different lengths, the maximization is done with the proper weighting (section length) of the data. In the process of developing the model, the study investigated different linear relationships in the exponential function. The final chosen model resulted in the best fit (maximized the likelihood). Furthermore, taking the natural logarithm of the pavement age, T , ensures that the DI at year zero is zero (CCI at year zero is 100), which is a desirable property. The AIC , which penalizes adding variables to the model, was used to validate incorporating the MSI in the model. Details of the AIC are presented in Appendix C.

Step 3. Empirical Bayes Estimate of Pavement Condition

Negative Binomial (Poisson-Gamma) Model

The Negative Binomial regression gives the coefficients of the model parameters (T , MSI , and intercept) that, when substituted in Equation 2 give DI_{model} , which is the average response of pavement sections with the same age T and MSI . Another parameter obtained from the Negative Binomial regression is what is referred to as the overdispersion parameter, ϕ . This parameter takes into account the variability of different pavement sections. For the Negative Binomial model, the variance, σ_s^2 , of the pavement sections condition can be calculated from DI_{model} and ϕ as shown in Equation 7. Under the Poisson error assumption, the variance of the error in condition reporting is equal to DI_{model} . One way to justify the dependence of the variance on the pavement condition is to consider that it is easier to rate pavements in good condition than it is to rate pavements in poor condition. This will lead to ratings of pavements in a poorer condition having higher variability (i.e., error). This seems plausible given that there

are several combinations of distresses that can cause a pavement section to be in poor condition but there is essentially one way (no distresses) for a pavement section to be in perfect condition. The data support this observation as more variability is observed for pavement sections that are in worse condition. Therefore, the total variance, σ_{mod}^2 , of the model can be calculated as shown in Equation 8.

$$\sigma_s^2 = \phi (DI_{\text{model}})^2 \quad (7)$$

$$\sigma_{\text{mod}}^2 = DI_{\text{model}} (1 + \phi DI_{\text{model}}) \quad (8)$$

The mean, DI_{model} , variance, σ_{mod}^2 , and overdispersion, ϕ , are related to the parameters of the Negative Binomial model. The Poisson-Gamma model gives rise to a Bayesian model with the Gamma distribution prior. In practical terms, DI_{model} is the prior, which represents the variability of the performance of different pavement sections. Once the data, DI , are observed, the posterior distribution of the true pavement condition, DI_{EB} , can be calculated using Bayes' formula. In the EB approach, the parameters of the prior are estimated from the data; in this case, by the Negative Binomial regression. The practical interpretation of Bayes' formula is that it combines the (practical) experience that can be learned from observing the historical performance of all pavement sections with specific observations to come up with an improved estimate of the pavement condition. The information from the prior and observation are combined using Equation 9. Note that Equation 3 is similar to Equation 9 with the added parameter α , which is included because the observed data do not strictly follow the Poisson-Gamma model.

$$DI_{EB} = \frac{1}{\phi DI_{\text{model}} + 1} DI_{\text{model}} + \left(1 - \frac{1}{\phi DI_{\text{model}} + 1} \right) DI \quad (9)$$

Deviation from the Negative Binomial (Poisson-Gamma) Model

The EB estimate in Equation 9 assumes the measurement error for reporting the *CCI* follows a Poisson distribution. The difference sequence method, which is described in Appendix D, was used to independently evaluate the variance of the error in reporting the *CCI* and it was found that this error variance is larger than what is predicted by the Poisson distribution. A concern could then be raised towards the applicability of the EB approach since the model assumptions, mainly Poisson error distribution, are violated. However, even if the Poisson-Gamma model is completely incorrect, as long as the variance of the error is not significantly overestimated, the EB estimate is still a better estimate than the actual measurement (either current or future). This is a result of linear Bayes estimators, which guarantee that irrespective of the true distribution of pavement performance, as well as the true distribution of the error in the measurement of the pavement condition, the linear Bayes estimator (of which the Poisson-Gamma model is one) improves on the estimate of the condition compared to just considering the measurement alone (see Hartigan, 1969, and Efron, 1973). The improvement of the linear Bayes estimator is such that the mean square error is reduced by a factor of:

$$\frac{\sigma_s^2}{\sigma_{\text{mod}}^2} = \frac{\sigma_s^2}{\sigma_s^2 + \sigma_{\text{Error}}^2} \quad (10)$$

If the error variance, σ_{Error}^2 , is underestimated, as is the case when assuming a Poisson error distribution, then the improvement of the EB estimate will be less than optimal. Therefore, the EB estimate in Equation 9 is conservative and can be improved if the appropriate value for the error variance is used. The linear EB estimator is calculated using Equation 11, which can be used for any two distributions and without knowledge of the appropriate distribution form:

$$DI_{\text{EB}} = \left(1 - \frac{\sigma_s^2}{\sigma_s^2 + \sigma_{\text{Error}}^2} \right) DI_{\text{model}} + \frac{\sigma_s^2}{\sigma_s^2 + \sigma_{\text{Error}}^2} DI \quad (11)$$

Note the similarities between Equation 9 and Equation 11 and it can be shown that if $\sigma_{\text{Error}}^2 = \alpha \sigma_{\text{Poisson}}^2$ (i.e., the error predicted from the Poisson distribution is different than the error in the data), then Equation 9 can still be used with ϕ replaced by $\phi_c = \phi/\alpha$, which is the form of Equation 3.

Step 4. Validating the Modeling Procedure

Dependence of Error in DI Measurement on Pavement Condition

Based on the difference sequence method, it was found that the Poisson distribution assumption underestimated the error variance of the observed data. A correction factor, α , was used to adjust for the discrepancy as shown in Equation 12. However, since for the Poisson distribution, the variance is equal to the mean, the measurement error was checked relative to the average of the observation as shown in Equation 13. Finally, the total variance of the data is related to the variance of the pavement sections' performance and the error variance as shown in Equation 14. The factor σ_{mod}^2 is the total variance, which should be equal to the variance of the observed data.

$$\sigma_{\text{Error}}^2 = \alpha \sigma_{\text{Poisson}}^2 \quad (12)$$

$$|\text{Error}_i|^2 \approx \alpha DI_{\text{model}}^i \quad (13)$$

$$\sigma_{\text{mod}}^2 = \sigma_s^2 + \sigma_{\text{Error}}^2 \quad (14)$$

Validating the Empirical Bayes Approach

The optimal test to validate a modeling procedure would be to know the actual true value that is being estimated (here the pavement deterioration represented by either the DI or the CCI) and verify that the chosen modeling procedure gives a better estimate of the true value compared

to no modeling (i.e., just using the observations). For real data, the true value is never known and this approach cannot be followed. An alternative approach is to compare the mean square error prediction with the criterion that the approach that results in a lower prediction error (of measurements not used in developing the model) is a better approach. We can think of the measurements used to estimate prediction error as measurements from future pavement surveys; clearly being able to better predict the pavement condition in future pavement surveys (for example next year's survey), is a desirable quality of any model. The prediction error was evaluated as follows:

1. Determine the prediction error for each pavement condition measurement as follows.
2. Remove the pavement condition measurement from the data and determine the year, Y , and section identifier, S , of that measurement.
3. Remove all pavement condition measurements from section S obtained after year Y . This step is essential because the problem of predicting next year's condition implies that we don't have access to the later years of conditions.
4. Estimate the condition for the removed measurement from the remaining set of measurements.

Five different approaches were evaluated to estimate the condition:

1. Most recent observation on the section S (Method 1): this is the simplest estimation method where we predict the condition as the most recent observation. The advantage of the method is that it takes into account the characteristics of the section S . The drawback of the method is that it assumes no deterioration within the year:

$$CCI^{i+1} = CCI^i$$

2. Model predicted condition (Method 2): this estimation method uses the fitted model to predict the condition:

$$CCI^{i+1} = CCI_{\text{model}}^{i+1}$$

3. EB estimate of most recent observation on the section S (Method31):

$$CCI^{i+1} = CCI_{EB}^i$$

4. Most recent observation on the section S with added deterioration estimated using the fitted model (Method 4):

$$CCI^{i+1} = CCI^i + (CCI_{\text{model}}^{i+1} - CCI_{\text{model}}^i)$$

5. EB estimate of most recent observation on the section S with added deterioration estimated using the fitted model (Method 5):

$$CCI_{EB}^{i+1} = CCI_{EB}^i + (CCI_{\text{model}}^{i+1} - CCI_{\text{model}}^i)$$

RESULTS AND DISCUSSION

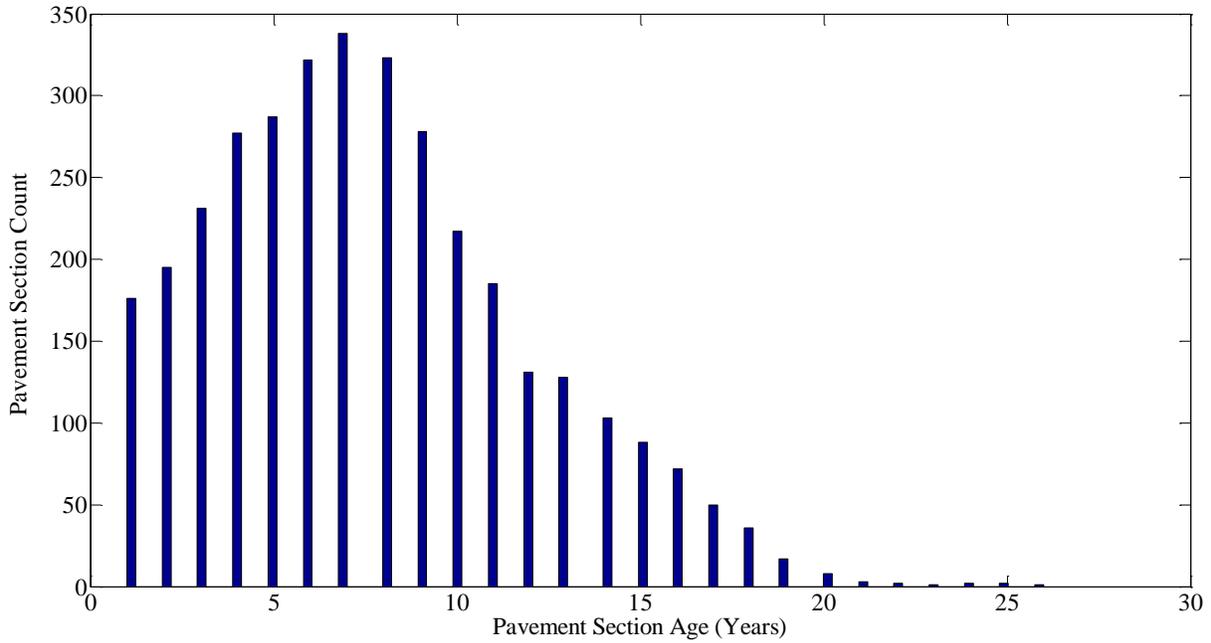
Step 1. Data Collection and Distribution of Pavement Condition

The collected data consist of section MSI, section condition (CCI), and section age (age since last recorded treatment). The distribution of the pavement sections' ages is presented in Figure 1. In total, there are 3,473 observations from 886 distinct pavement sections with 7 years being the most observed age. Figure 2 shows the distribution of the MSI. In total, 39.5% of pavement sections had MSI values lower than one. Figure 3a shows the CCI values as a function of pavement age. The mean of each age group with the two standard deviation range are also shown in the figure. Clearly, the two standard deviation range (which is used to approximate the 95% confidence interval of normally distributed data) does not give an adequate representation of the data as it extends beyond the CCI limit of 100. Figure 3b shows a box and whisker plot with the box representing the data between the 25th and 75th percentile and the whiskers extending to a maximum of 1.5 times the box range. Data points that fall outside the maximum whiskers range are labeled with a "+" symbol.

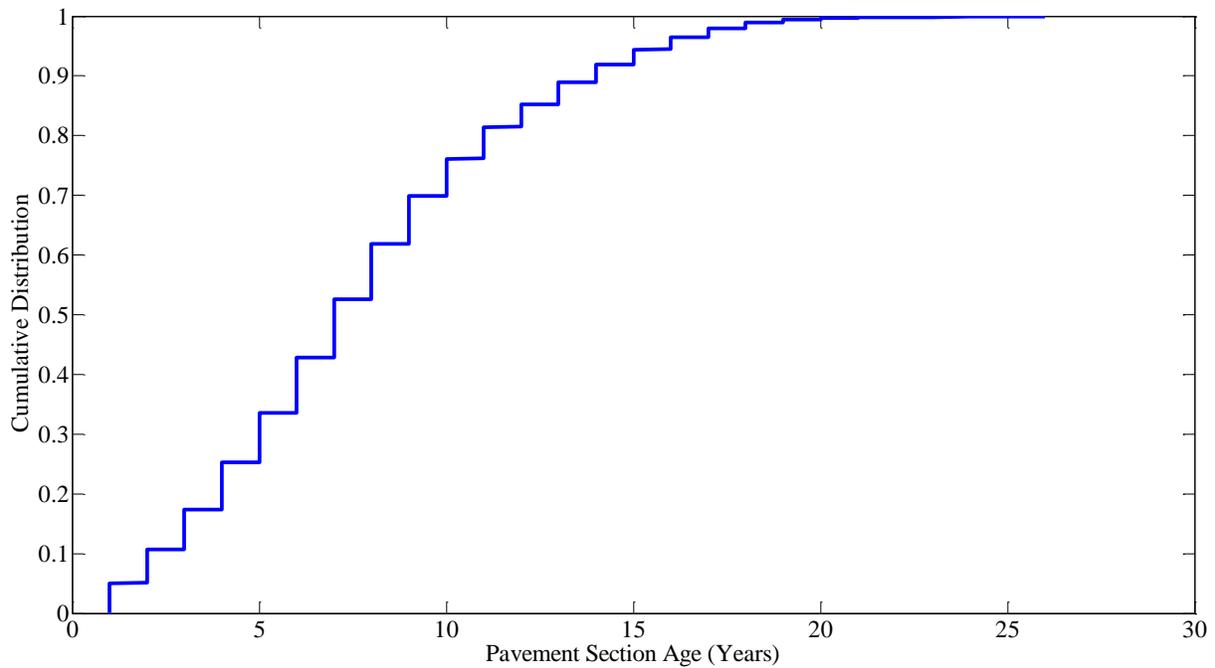
Figure 4 shows the distribution of the DI for all the data with different fitted theoretical distributions; mainly, a Poisson distribution, a Normal distribution, a Negative Binomial distribution, and a mixture of three Negative Binomial distributions. The Poisson distribution is a discrete probability distribution for positive values and was considered because the DI values are discrete and positive and combined with the Gamma distribution results in the Negative Binomial distribution. The Poisson is restricted to having a variance equal to the mean of the distribution, which results in a poor representation of the data. The Normal distribution allows the variance to be independent of the mean and gives a better representation of the data; however, it is not a discrete probability distribution, the fit is still not very good, and more important, it allows for negative DI values, which is not realistic. The Negative Binomial distribution gives a better fit to the data and is restricted to positive DI values.

Figure 5 shows the Negative Binomial Distribution fitted to DI observations obtained at a fixed age (ages of 1, 4, 7, and 10 are shown). The fits for pavement ages 1, 4, and 7 years are almost perfect. For the 10-year pavement age, the observed data deviate from the Negative Binomial distribution, although it is still reasonable for modeling purposes (at least much more reasonable than the Normal distribution assumption). There is, however, a very plausible explanation for the lack of fit of the Negative Binomial distribution as the age increases. The main purpose of a PMS is to keep the roads at an acceptable level of service. When a pavement reaches an unacceptable condition level, action is taken to bring the condition back to acceptable levels. This type of action biases the data, as only pavements that perform well are allowed to reach a higher age. This means that if the condition of pavement sections at a given year, say 15

years, follows a certain distribution (in this case the Negative Binomial distribution), the observed data will deviate from that distribution because the poorly performing pavements will not be observed at 15 years because they would have already been treated.

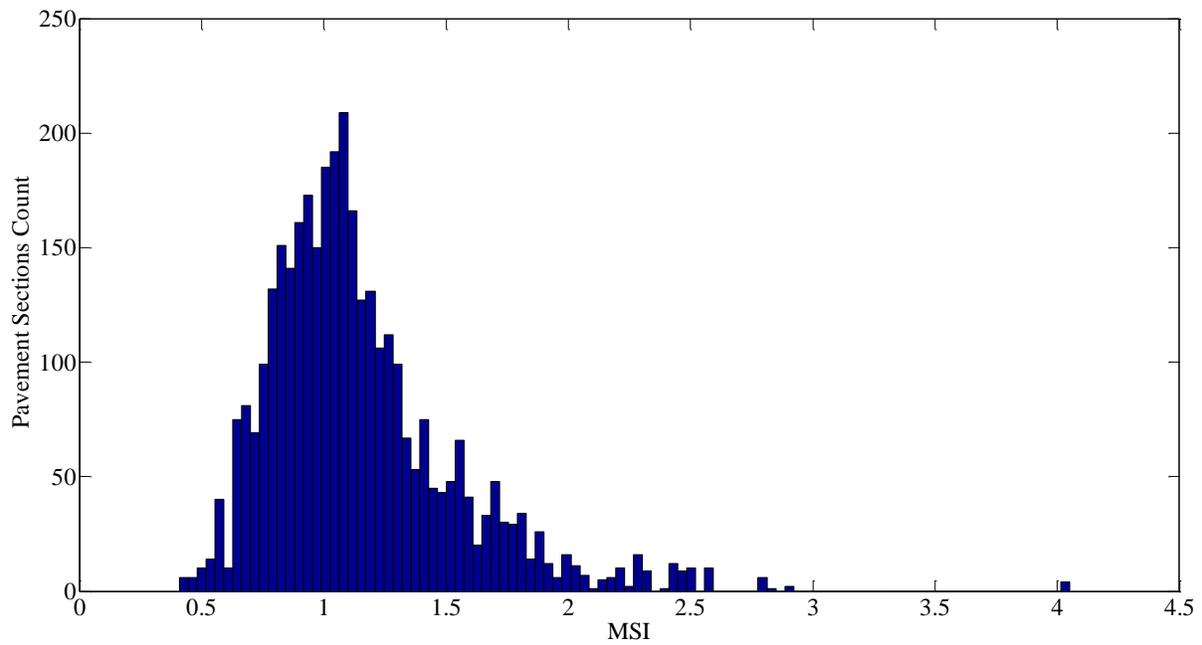


(a)

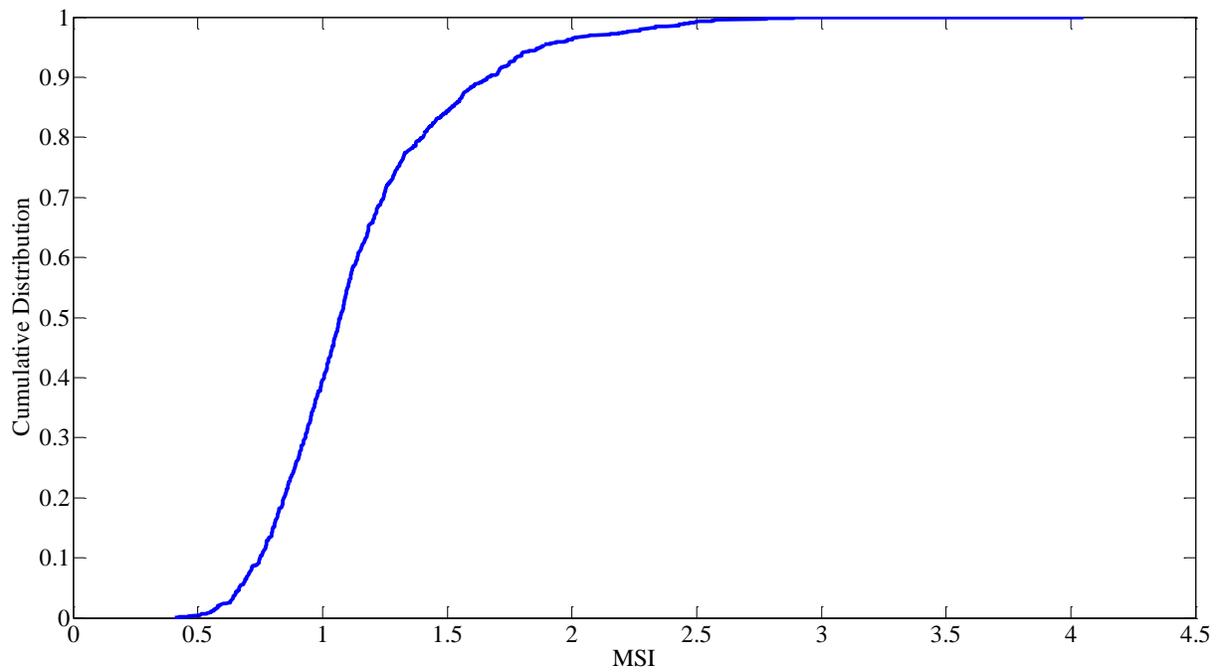


(b)

Figure 1. Distribution of Pavement Section Ages; the age is determined from the last recorded pavement treatment: a) Histogram; b) Cumulative Distribution

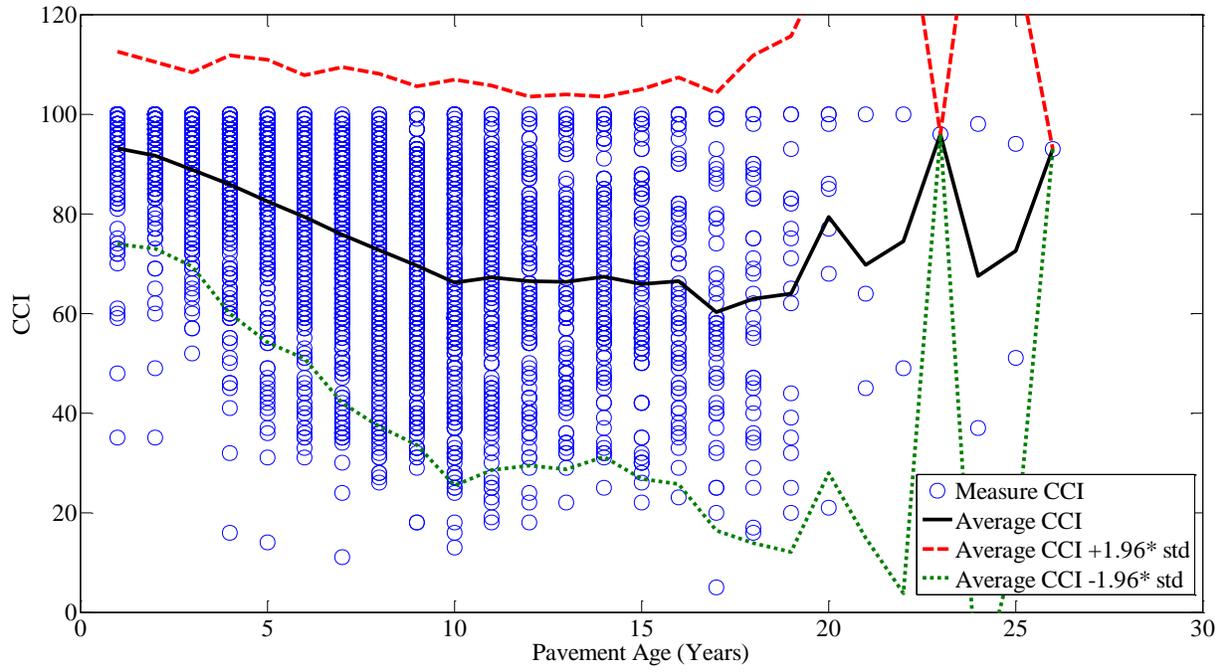


(a)

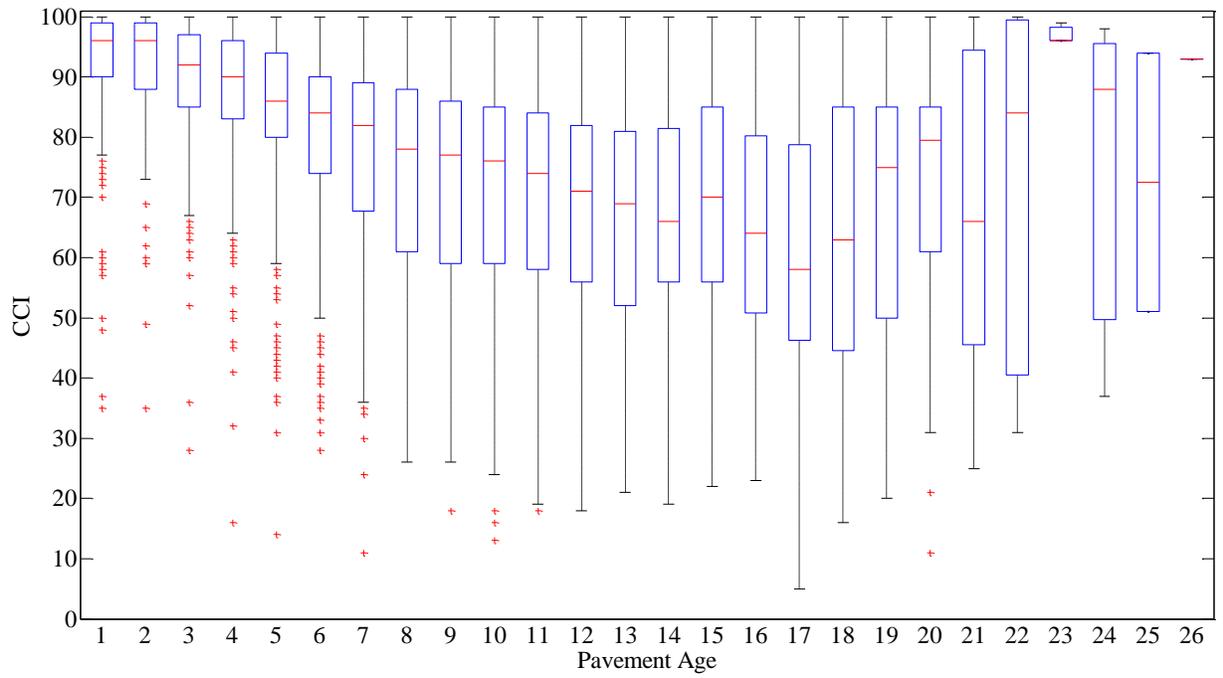


(b)

Figure 2. Distribution of MSI Values: a) Histogram; b) Cumulative Distribution



(a)



(b)

Figure 3. Pavement CCI as a Function of Age: a) Scatter Plot; b) Boxplot

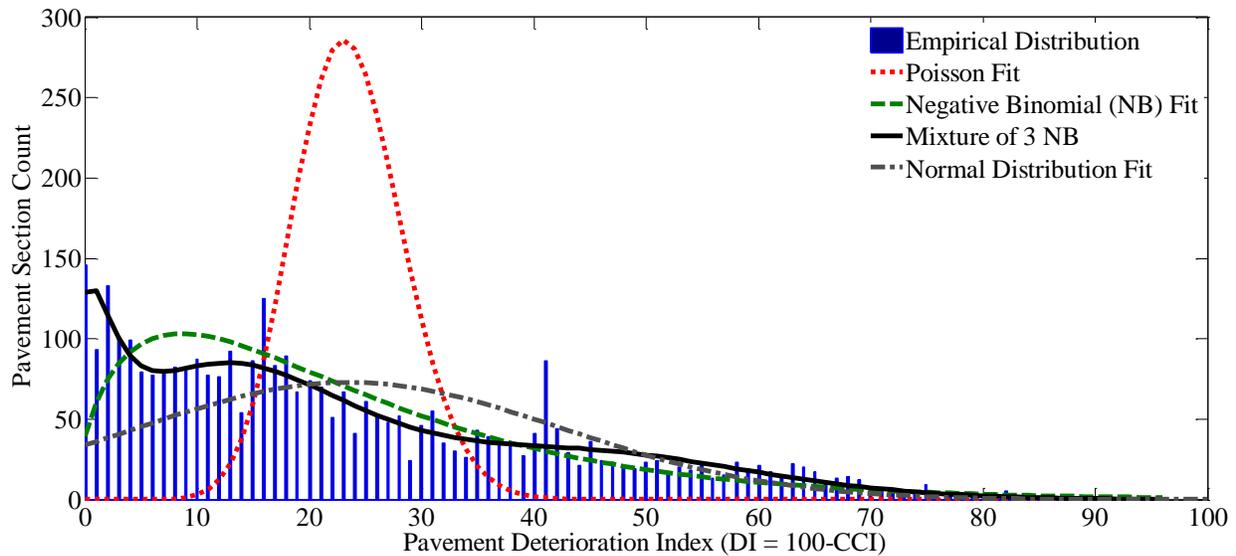


Figure 4. Distribution of Pavement DI

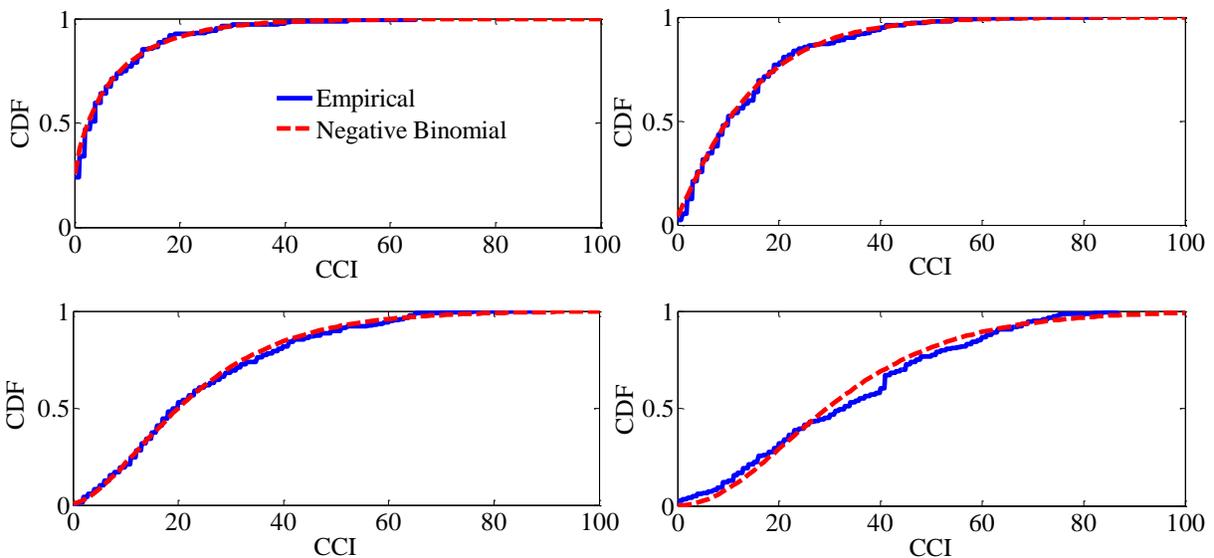


Figure 5. Comparison of Empirical Distribution of the Pavement CCI to Negative Binomial Distribution Fit at Different Ages; Top Left: 1 Year; Top Right: 4 Years; Bottom Left: 7 Years; Bottom Right: 10 Years

Figure 3 provides supporting evidence for this explanation. From years 1 to 10, the average *CCI* decreases with increasing age as would be expected. However, from year 11 onwards, the average *CCI* practically stays at the same level as year 10 (after year 16, the *CCI* varies significantly due to the limited data). This contradicts the common engineering sense, as it is known that pavements will continue to deteriorate. The explanation in this case is that poorly performing pavements are treated before reaching year 11; the average *CCI* for pavements 11 years and older is a biased representation of the performance of all pavements as only good performing pavements are allowed to reach that age. To reduce the biasing effect that pavement treatment has on model estimation, the regression model was fitted to data from

observations of pavement sections that had the last treatment performed less than 10 years prior to the observation.

Step 2. Deterioration Model Development

The estimate of the parameters of the model (Equation 2) were $\beta_0 = 1.7027$, $\beta_1 = 0.0490$, $\beta_2 = 0.0866$, and $\beta_3 = 0.1595$, as obtained from the weighted Negative Binomial regression with the data limited from 1 to 10 years (representing 76% of all the data). The overdispersion parameter ϕ was equal to 0.3135. The AIC weight, w , for the model with MSI and the model without MSI was less than 2×10^{-5} indicating that the model with the MSI is at least 50,000 times more likely to be closer to the true pavement deterioration than the model without the MSI.

Deterioration curves for different MSI values are shown in Figure 6. Note that for MSI larger than 1, the predicted performance is practically the same (for MSI large than 1, it is hard to distinguish the different deterioration curves in the graph) whereas the performance changes significantly as the MSI decreases below 1. The limit for MSI increasing to infinity reflects the fact that pavement deterioration is not solely dependent on the MSI; other factors, such as environmental loading and top-down cracking, also play a role in pavement deterioration and these factors lead to deterioration no matter how strongly a pavement is designed.

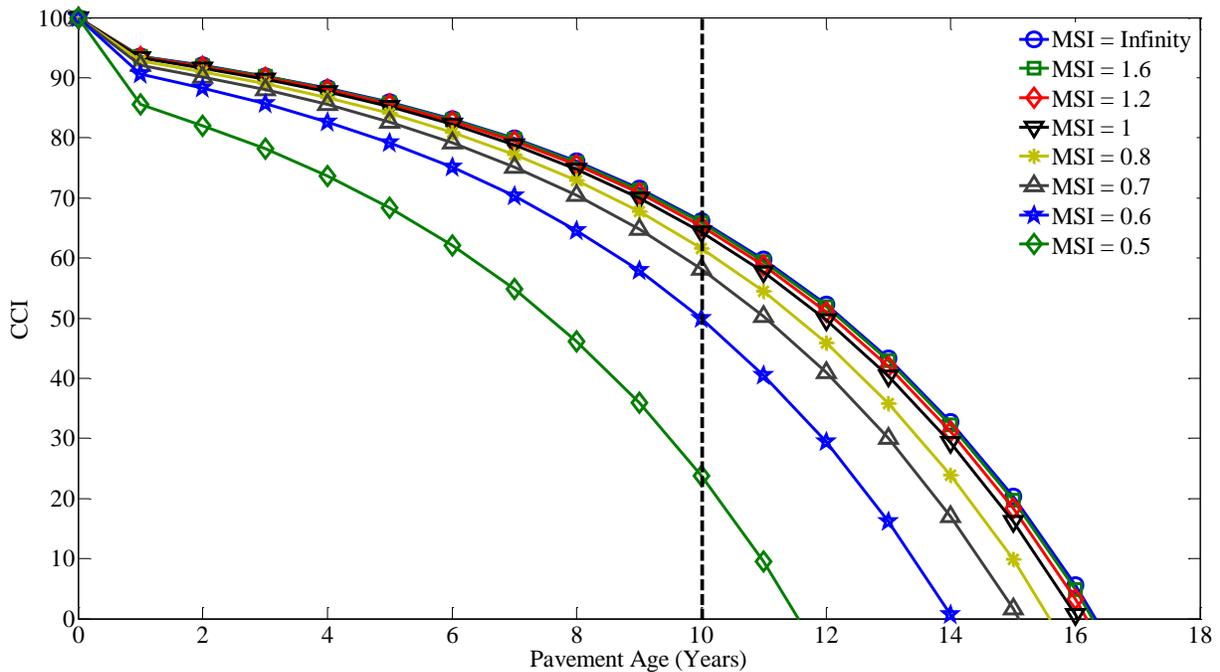


Figure 6. Pavement Deterioration for Different MSI values. The vertical dotted line indicates the last year of data used to fit the model (i.e., model predictions after year 10 are based on extrapolation and therefore should be used with caution).

Step 3. Empirical Bayes Estimate of Average Pavement Condition

The EB approach combines the model estimate with the individual observations of DI , which does improve on the individual estimate of the pavement condition C . The EB estimated condition takes into account the variability of the results of the pavement condition survey along with the variability of the performance of individual pavement sections that is not taken into account by the model to obtain a more accurate estimate of the pavement condition. The better accuracy can be inferred from the fact that the EB estimate results in better prediction of future pavement condition, which is presented at the end of this section.

EB Estimate of Pavement Condition Data

Figure 7 shows for every observation, the recorded CCI , CCI_{model} , and CCI_{EB} . The net effect of the EB methodology is to pull the observed CCI toward CCI_{model} . Figure 8 shows the difference between the measured DI and DI_{EB} .

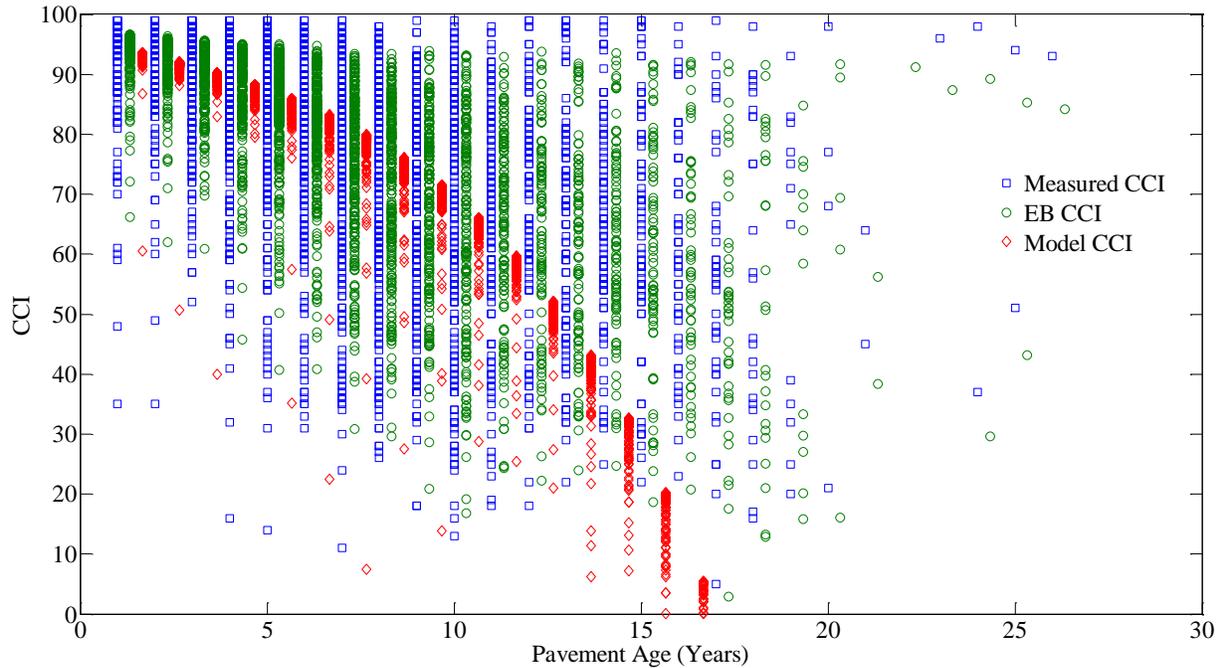


Figure 7. Comparison of Measured CCI , Model Predicted CCI , and EB Estimate of CCI . The EB estimate combines the measured CCI with the model predicted CCI to obtain a better estimate of the “true” CCI . The plots of the EB CCI and the model CCI are shifted to visually distinguish between the different estimates of the CCI .

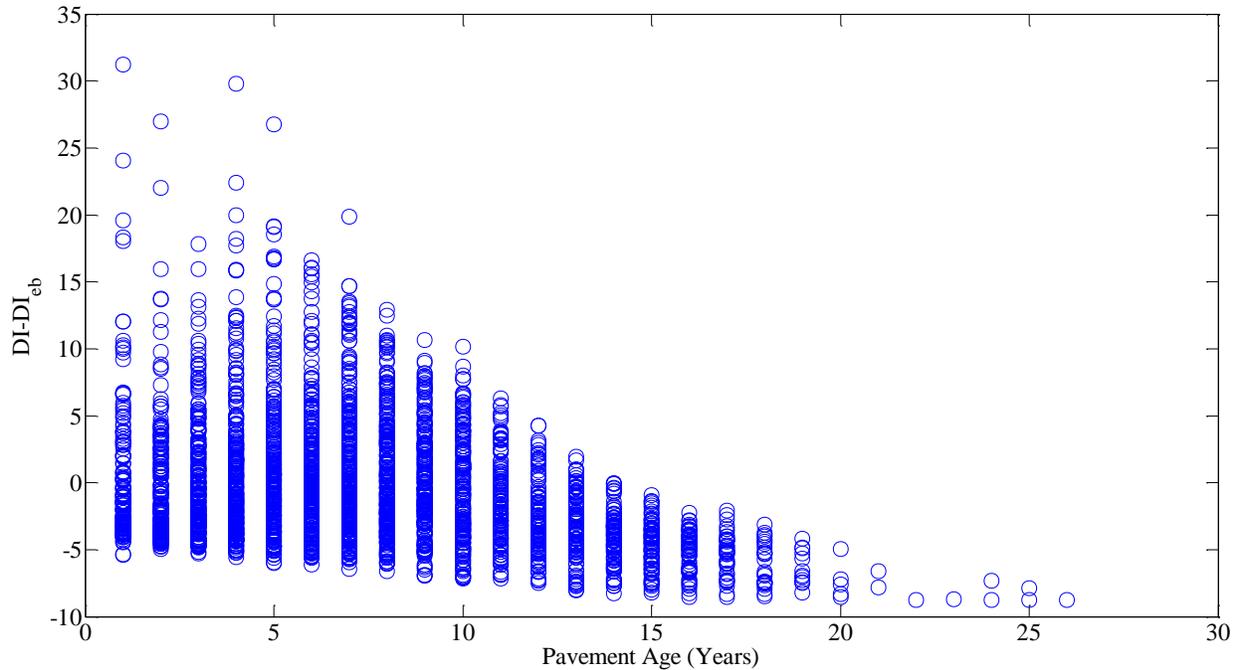


Figure 8. Difference between Measured Pavement Condition and EB Estimated Pavement Condition

The benefits of the EB estimate is that it is a better estimate of the true pavement condition than either the recorded *CCI* from the condition survey data or the condition predicted from the developed deterioration model. The criterion used to determine which method gives a better estimate of the true condition is prediction error (estimates that have lower prediction error are closer to the true value than estimates that have higher prediction error). The reason why the EB estimate is better is because it reduces some of the variability in the condition survey data. Five methods to predict pavement condition were evaluated, and the mean square prediction error of each method is shown in Table 1. Method 2, which uses the developed regression model, gives the worse prediction error. This is because even after taking into account age and structural condition, which are incorporated in the model, there is a significant variation in the deterioration of different pavement sections. The model does not reflect the deterioration of each specific pavement section; it represents only the “average” behavior of all pavement sections and the observation from a specific section (Method 1) is a better estimate to predict the following year’s condition. Method 3 is the empirical Bayes estimate, which linearly combines (using a weighted average) the individual *CCI* values with the estimate from the regression model. This combination improves the prediction of future condition. Method 4 uses the individual *CCI* values along with the expected deterioration calculated from the regression model. The performance of this method is slightly better (practically the same) than Method 3. Method 5 uses both the EB estimate and the expected deterioration calculated from the regression model (it can be seen as combining Method 3 and Method 4). This method gives the lowest prediction error, which is significantly better than all other methods and 21.6% better than the estimate of Method 1, which does not use any modeling of pavement deterioration.

Table 1. Mean Square Error (MSE) of Prediction Using the Different Methods to Estimate Future Pavement Condition

Method	MSE Prediction	MSE Ratio with Method 1
1	143.2	1.000
2	227.0	1.585
3	130.9	0.914
4	129.2	0.902
5	112.2	0.784

Step 4. Evaluating Model Assumptions

For two independent Poisson-distributed random variables with means λ_1 and λ_2 , the distribution of the difference ($\lambda_2 - \lambda_1$) is given by the Skellam distribution (Skellam, 1946). Figure 9 shows the empirical distribution of the difference between two consecutive observations of the *DI*, the theoretical distribution based on the Poisson-Gamma model, and the best-fit Normal distribution. Clearly the Normal distribution is not representative of the behavior of the data. The Skellam distribution gives a much better representation. The fit is, however, not perfect as the data exhibit more outlying observations than what is predicted by the model. The difference between the observed empirical distribution and the theoretical Skellam distribution can be quantified with the standard deviation. The standard deviation of the observed data is 12.4 while the standard deviation of the theoretical distribution is 7.6, which underestimates the spread in the data. Underestimating the standard deviation makes the EB results conservative. That is to say, the results can be improved if the procedure is adjusted to better reflect the true standard deviation. The ratio of the standard deviations is 1.63 (12.4/7.6), which makes the ratio of the variances equal to 2.66.

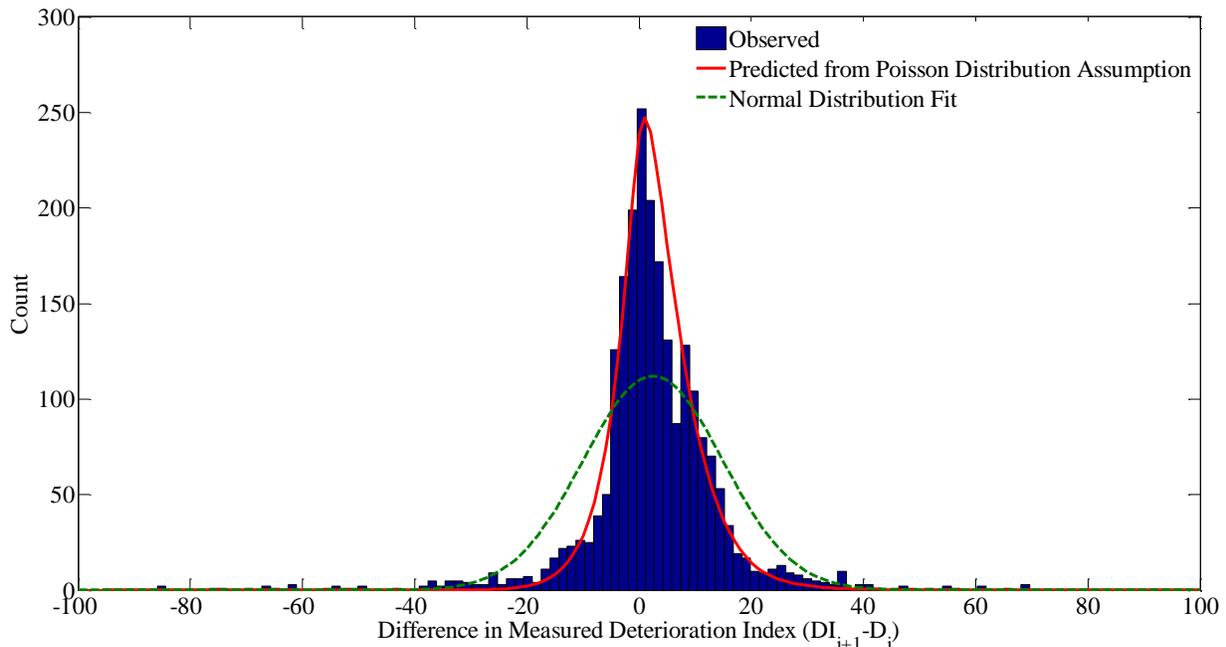


Figure 9. Comparison of the Empirical Distribution of the Difference Between Consecutive *DI* Values With Model Predicted Distribution and Best Fit Normal Distribution

Figure 10 shows that the estimated measurement error increases as a function of the estimated measurement condition. For the Poisson distribution, the error variance is equal to the condition. A regression line with a zero intercept was fitted to the square of the error shown in Figure 10. The slope of the fitted line was found to be equal to 2.88, which suggests that the error is not Poisson-distributed, with a variance larger than what is predicted by the Poisson distribution. However, similar to the Poisson distribution, the error is linearly related to the condition. The two factors of 2.66 and 2.88 are estimates of the deviation of the error variance from the Poisson distribution variance. However, both estimates have drawbacks. The estimate of 2.66 obtained from the comparison to the Skellam distribution includes the variance pavement deterioration and therefore is larger than the error variance. The estimate of 2.88 obtained from the regression performed on the estimated square error uses all observations. These include possible treatments that were not recorded, which inflate the estimated error variance. The final estimate was obtained from observations that had a positive change in the *DI*, which precludes those sections missing a record of treatment that occurred. The factor calculated from those observations, α was 2.59, meaning that the error variance is 2.59 times the error variance predicted from the Poisson distribution.

The final verification consists of checking whether Equation 14 is satisfied; σ_s^2 was calculated as 190 while σ_{Error}^2 was calculated as 22 for a total σ_{mod}^2 of 212. The total variance of the data was estimated as 273, which shows that the model variance underestimates the data variance by 61 (273-212). The estimate of the error variance of the data was obtained for the positive error because of possibly missing treatment records. However, the total variance of the data includes observations with possibly missing records of treatment. Therefore the variance of all the errors (positive and negative) was evaluated as 72 and added to σ_s^2 to give a total variance for the model $\sigma_{mod}^2 = 262$, which is reasonably close to the total variance of 273 of the data. This shows that the modeling approach is consistent, as the resulting model accounts for practically all the variance observed in the data.

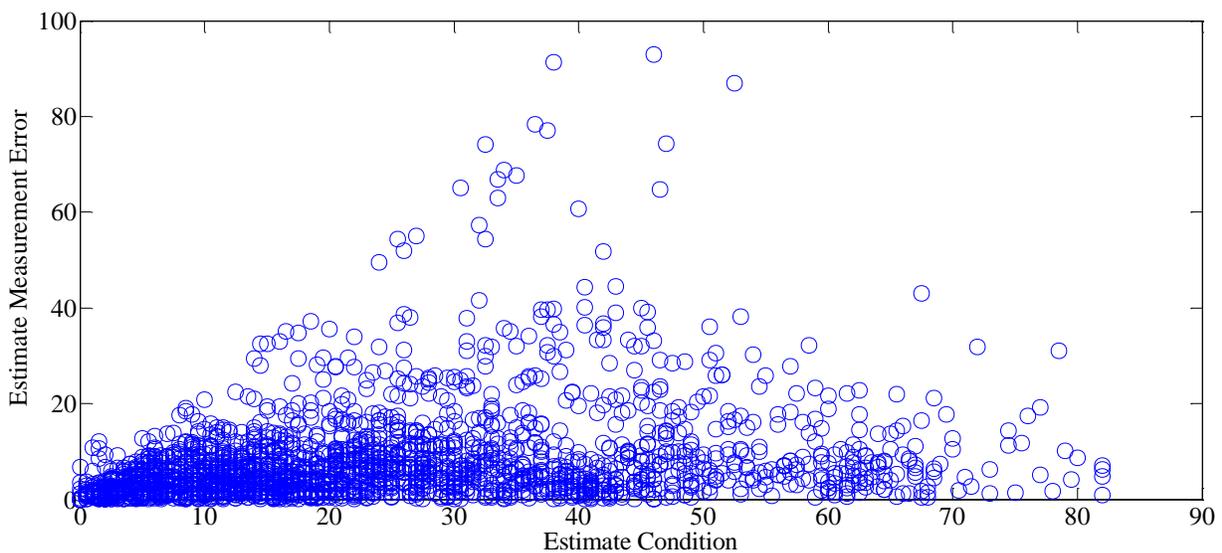


Figure 10. Measurement Error as a Function of Pavement Condition

CONCLUSIONS

- *The pavement structural condition (MSI parameter) is a significant parameter that affects the pavement condition:* using the AIC criterion, the model that incorporates the pavement structural parameter as an explanatory variable of the pavement condition was more than 50,000 times more likely than the model that does not incorporate the pavement structural condition.
- *The Negative Binomial distribution gives a good representation of the pavement condition:* this allows for a better understanding and modeling approach to pavement condition where variability in pavement condition can be decomposed into variability due to different performance of different pavement sections and variability due to error in measuring the pavement condition. The resulting model can be used for network-level pavement management.
- *Condition of pavement sections older than 10 years do not represent the typical (expected) performance of pavement sections:* pavement sections are rehabilitated once they reach an unacceptable condition level. Most pavement sections need some sort of treatment before 11 years have passed since the last treatment and only pavement sections that perform well reach 11 years of age or more. Therefore, those pavement sections give a biased representation of pavement condition after 10 years.
- *The optimal estimate of the pavement condition is one that combines the observed condition with the model predicted condition:* the estimate is obtained by an EB approach, which combines the model estimate with the observed condition through a weighted average. The weight is determined by the relative variability of the error in the measurement of the pavement condition and the variability of the performance of different pavement conditions. The model on its own gives an inaccurate estimate of the pavement condition with a mean square error that is about 1.58 times the mean square error prediction of future observations. However, combining the observations with the model resulted in an estimated mean square error prediction that is about 0.78 times the mean square error prediction of the observations. The estimate of the improvement is based on cross-validation where observations are held out and used to estimate the mean square error prediction.

RECOMMENDATIONS

1. *VDOT's Maintenance Division should implement the empirical Bayes method to determine the pavement condition of Interstate roads:* the EB method was found to improve the prediction of future CCI measurements by an estimated average of 21.6%.
2. *VDOT's Maintenance Division should develop a similar approach for the pavement condition of primary and high-volume secondary roads:* the implementation for secondary roads that are only evaluated at 5-year cycles is especially needed, and for this purpose FWD data collection at the network level is suggested for these roads. The EB method combined

with the modeled deterioration should provide for a better prediction of the conditions of secondary roads during the years when the condition is not collected.

3. *VDOT's Maintenance Division should continue performing network-level pavement structural evaluation:* The pavement structural condition summarized in terms of the MSI was found to affect the rate of pavement deterioration.

BENEFITS AND IMPLEMENTATION

Benefits

The primary benefit of this study to VDOT is that VDOT will be able to predict more accurately the future condition of pavement sections on its network using the empirical Bayes approach. Although no direct cost savings are anticipated, improved predictions should support more efficient use of resources through VDOT's needs-based budgeting process.

The empirical Bayes (EB) approach can further improve the estimate of the pavement condition. The proposed approach can improve the mean square error prediction of the future (next year's) pavement condition by 21.6%. This improved estimate of the future pavement condition is expected to minimize the difference between network-level planning and project-level treatment selection, which will result in more effective management of the pavement assets.

Implementation

With regard to Recommendations 1 and 2, VDOT's Maintenance Division will work in cooperation with VDOT's Information Technology Division to implement the suggested methodology within the PMS and apply these steps wherever recent and reasonable data are available. It is expected that this will have an impact on the results of the network level analysis; maintenance and rehabilitation recommendations from the PMS; and budgetary needs and other reports currently prepared by VDOT. VDOT's Maintenance Division should further perform sensitivity analyses on the final results and recommend changes to the current methodologies and allocation wherever applicable.

With regard to Recommendation 3, VDOT plans to continue collecting network level structural condition data. VDOT is a participating agency in Transportation Pooled Fund Study TPF-5(282), Demonstration of Network Level Pavement Structural Evaluation with Traffic Speed Deflectometer. This pooled fund study is anticipated to be complete by the end of the fourth quarter of calendar year 2016 and will provide suggestions to VDOT as to how best to accomplish this testing.

ACKNOWLEDGMENTS

The authors acknowledge the contributions of the project's technical review panel: Tanveer Chowdhury, David Shiells, and Bryan Smith. The authors also thank Raja Shekharan and Akyiaa Morrison from VDOT for their review of, and input to, the project report.

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APPENDIX A

MODIFIED STRUCTURAL INDEX METHODOLOGY

This appendix presents the methodology to calculate the modified structural index (MSI) from FWD data and describes how to update the MSI based on applied pavement treatment.

Calculating the Modified Structural Capacity Index (MSI)

The MSI is defined in Equation A.1

$$MSI = \frac{SN_{eff}}{SN_{req}} \frac{0.4728 (D_0 - D_{1.5H_p})^{-0.4810} H_p^{0.7581}}{0.05716 [\log(ESAL) - 2.32 \log(M_r) + 9.07605]^{2.36777}} \quad (A.1)$$

where

D_0 = the FWD deflection under the applied load

H_p = total pavement depth (i.e., measured from the top of the pavement to the top of the subgrade)

$D_{1.5H_p}$ = FWD deflection at a distance equal to 1.5 times the total pavement depth

M_r = resilient modulus calculated using FWD measurements

$ESAL$ = total accumulated truck traffic over a total design period of 20 years.

Detailed steps to calculate of $D_{1.5H_p}$, M_r , and $ESAL$ are as follows:

1. The FWD measurements should be normalized to 9,000 lb load deflections.
2. The deflections at an offset of 1.5 times the total pavement depth (from top of surface to top of subgrade) are calculated using the following interpolation shown in Equation A.2:

$$D_{1.5H_p} = \frac{(x-B)(x-C)}{(A-B)(A-C)} D_A + \frac{(x-A)(x-C)}{(B-A)(B-C)} D_B + \frac{(x-A)(x-B)}{(C-A)(C-B)} D_C \quad (A.2)$$

where x is the distance from the applied load to 1.5 times the depth of the pavement (H_p), A , B and C are the three points where the deflection is measured from the FWD that are closest to x such that $A < x < C$ (and $B < x$ or $x < B$ are valid configurations), and D_A , D_B , and D_C are the deflections at points A , B and C respectively (see Figure A.1).

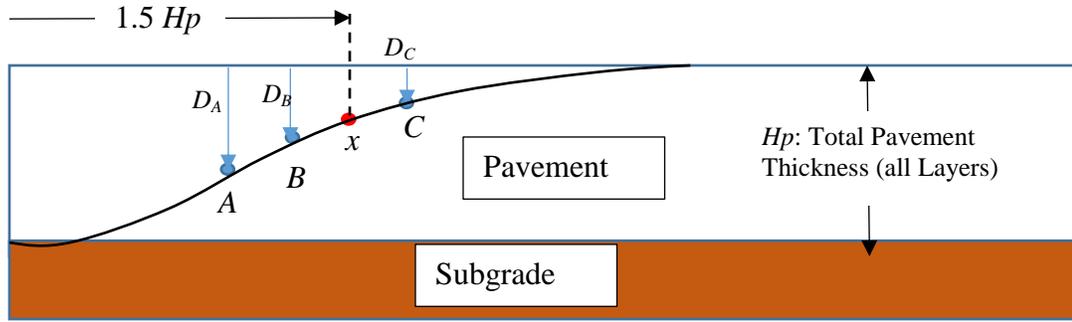


Figure A.1. Schematic for the Calculation of $D_{1.5H_p}$

3. Estimate the design resilient modulus from the FWD measurements using Equation A.3:

$$M_r = \frac{0.24C \times P}{D_{1.5H_p} \times (1.5H_p)} \quad (\text{A.3})$$

where $C = 0.33$ (AASHTO, 1993), P is the applied load in pounds.

4. Calculate the ESAL using Equation A.4:

$$ESAL = AADTT \times T_f \times G \times D \times L \times 365 \times Y \quad (\text{A.4})$$

where

$AADTT$ = average annual daily truck traffic

T_f = truck factor for flexible pavements obtained from Table A.1

G = growth factor set to 1.3435 (3% annual growth over a 20 years period)

D = directional factor set at 0.5

L = lane factor set at 0.9

Y = is the number of years in the design period set to 20.

Table A.1. Statewide Average Truck ESAL Factors (from Smith and Diefenderfer, 2009)

Pavement Type	Single-Unit Trucks	Combination Trucks
Flexible	0.46	1.05
Rigid	0.59	1.59

Updating the Modified Structural Capacity Index (MSI) Based on Pavement Treatment

The MSI for flexible pavements is based on the ratio of the required structural number (SN_{Req}) to the effective structural number and (SN_{Eff}). Therefore, any significant maintenance actions are expected to affect (increase) the SN_{Eff} . In order to update the MSI value, equation 5 may be used when the depth of the maintenance does not exceed the depth of the asphalt layers. If the maintenance exceeds the depth of the asphalt layers, the numerator in equation A.5 should be replaced by the SN_{Eff} from the pavement design documents.

$$MSI_{Updated} = MSI_0 + \frac{0.44(d_{placed} - c \times d_{milled})}{0.05716(\log(ESAL) - 2.32\log(M_r) + 9.07605)^{2.36777}} \quad (A.5)$$

where $MSI_{Updated}$ is the updated value for the MSI, MSI_0 is the original value of the MSI, d_{placed} is the depth of the asphalt layer (in inches) placed, d_{milled} is the depth of the milled asphalt layer (in inches), and c is a factor based on the condition of the pavement (Huang 2004).

Recommended values for c are as follows:

$c = 1.0$ for existing pavement in good overall structural conditions with little or no cracking

$c = 0.75$ for existing pavement with initial transverse and corner cracking due to loading but without progressive structural distress or recent cracking

$c = 0.35$ for existing pavement that is badly cracked or shattered structurally.

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APPENDIX B

PROBABILITY DENSITY FUNCTIONS

This appendix gives the probability density functions for the Negative Binomial Distribution, the Poisson distribution, and the Gamma distribution given in Equation B.1, Equation B.2, and Equation B.3, respectively. An alternative parametrization of the Gamma distribution is given in Equation B.4. The definition of the Negative Binomial distribution in terms of the Poisson and Gamma distributions is given in Equation B.5.

$$f_{NB}(x; r, p) = \frac{\Gamma(r+x)}{x! \Gamma(r)} p^x (1-p)^r \quad (\text{B.1})$$

$$f_P(x; \lambda) = \frac{\lambda^x}{x!} e^{-\lambda} \quad (\text{B.2})$$

$$f_G(\lambda; r, p) = \frac{\lambda^{r-1}}{\left(\frac{p}{1-p}\right)^r \Gamma(r)} e^{-\lambda(1-p)/p} \quad (\text{B.3})$$

$$f_G(x; r, \theta) = \frac{x^{r-1}}{\theta^r \Gamma(r)} e^{-\frac{x}{\theta}} \quad (\text{B.4})$$

$$f_{NB}(x; r, p) = \int_0^\infty f_P(x; \lambda) f_G(\lambda; r, p) d\lambda \quad (\text{B.5})$$

APPENDIX C

MODEL FITNESS

The *AIC* assesses the fitness of a model based on the log-likelihood value of the model, L , and a penalty term related to the number of parameters, p . The *AIC* is calculated as shown in Equation C.1:

$$AIC = -2\ln(L) + 2p \quad (C.1)$$

The *AIC* does not give an indication whether the model is the true model that generated the data. It can only be used to compare models and evaluate which one is more likely to be closer to the true model. This is done by calculating the exponential of half the relative difference between the *AIC* of two models being considered as given in Equation C.2

$$w = \exp\left(\frac{AIC_{\min} - AIC_2}{2}\right) \quad (C.2)$$

where w is the relative likelihood of model 2, compared to the model with the lowest *AIC* (model 1), of being the model closer to the true (unknown) model that generated the data compared to the model with lowest *AIC* (Burnham and Anderson, 2004).

The two models evaluated in this report are the model with only the pavement age as a predictor of pavement condition and the model with pavement age and MSI as predictors of pavement condition.

APPENDIX D

DIFFERENCE SEQUENCE METHOD

The difference sequence method estimates the error standard deviation by taking the difference between consecutive measurements. Given the pavement condition of two consecutive years DI_i and DI_{i+1} , the difference can be calculated as follows

$$DI_{i+1} - D_i = C_{i+1} + \varepsilon_{i+1} - (C_i + \varepsilon_i) = C_{i+1} - C_i + \xi_{i+1} \quad (D.1)$$

where C represents the pavement condition (without error in the measurement) and ε is the measurement error and ξ is the difference in the error. If ε has variance σ_{Error}^2 then ξ has variance $2\sigma_{Error}^2$. In general $C_{i+1} - C_i \ll \xi_{i+1}$ and therefore,

$$DI_{i+1} - D_i \approx \xi_{i+1} \quad (D.2)$$

This approximation can be improved by noting that the deterioration predicted by the model can be used to estimate $C_{i+1} - C_i$ as follows

$$DI_{\text{model}}^{i+1} - DI_{\text{model}}^{i+1} \approx C_{i+1} - C_i \quad (D.3)$$

Therefore,

$$DI_{i+1} - D_i - (DI_{\text{model}}^{i+1} - DI_{\text{model}}^{i+1}) \approx \xi_{i+1} \quad (D.4)$$

and Equation D.4 is a better estimate of ξ than Equation D.2 and can be used to estimate the measurement error variance σ_{Error}^2 .

APPENDIX E

SKELLAM DISTRIBUTION AND BESSEL FUNCTION

The Skellam distribution is given in Equation E.1:

$$f_S(x; \lambda_1, \lambda_2) = e^{-(\lambda_1 + \lambda_2)} \left(\frac{\lambda_2}{\lambda_1} \right)^{\frac{x}{2}} I_x(2\sqrt{\lambda_1 \lambda_2}) \quad (\text{E.1})$$

where I_x is the modified Bessel function of the first kind given by Equation E.2

$$I_x(y) = \sum_{m=0}^{\infty} \frac{1}{m! \Gamma(m + \alpha + 1)} \left(\frac{y}{2} \right)^{2m + \alpha} \quad (\text{E.2})$$

APPENDIX F

EMPIRICAL BAYES APPLICATION

This appendix describes application of the empirical Bayes (EB) method using the measured CCI and the CCI obtained from the regression model CCI_{model} to obtain CCI_{EB} .

Input:

$$CCI = 45; T = 7 \text{ years}; MSI = 0.7$$

$\beta_0 = 1.7027$, $\beta_1 = 0.0490$, $\beta_2 = 0.0866$ and, $\beta_3 = 0.1595$, $\phi = 0.3135$ and $\alpha = 2.59$ (these were determined from the PMS data)

Solution:

$$\text{Calculate } DI = 100 - CCI$$

$$DI = 100 - 45 = 55$$

$$\text{Calculate } DI_{\text{model}}$$

$$DI_{\text{model}} = e^{\left(\beta_0 + \beta_1 \frac{1}{MSI^4} + \beta_2 \ln(T) + \beta_3 T\right)} = T^{\beta_2} e^{\left(\beta_0 + \beta_1 \frac{1}{MSI^4} + \beta_3 T\right)}$$

$$DI_{\text{model}} = e^{\left(1.7027 + 0.0490 \frac{1}{0.7^4} + 0.0490 \ln(7) + 0.1595 \cdot 7\right)} = 22.6$$

$$\text{Calculate } DI_{EB}$$

$$DI_{EB} = \frac{1}{\frac{\phi}{\alpha} DI_{\text{model}} + 1} DI_{\text{model}} + \left(1 - \frac{1}{\frac{\phi}{\alpha} DI_{\text{model}} + 1}\right) DI$$

$$DI_{EB} = \frac{1}{\frac{0.3135}{2.59} \times 22.6 + 1} \times 22.6 + \left(1 - \frac{1}{\frac{0.3135}{2.59} \times 22.6 + 1}\right) \times 55 = 46.3$$

$$\text{Calculate } CCI_{EB}$$

$$CCI_{EB} = 100 - CCI_{EB} = 100 - 46.3 = 53.7$$